



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>



3 3433 06910023 2



PAM
Wendell

A MANUAL OF PHYSICAL MEASUREMENTS

BY
GEORGE V. WENDELL, PH.D. 1911-
PROFESSOR OF PHYSICS, COLUMBIA UNIVERSITY

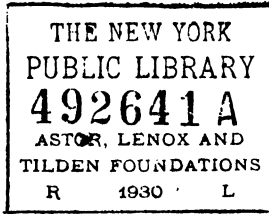
AND

WILLARD L. SEVERINGHAUS, PH.D. 1902-
INSTRUCTOR IN PHYSICS, COLUMBIA UNIVERSITY

FOR THE USE OF STUDENTS IN COLUMBIA COLLEGE
PREPARING FOR ADMISSION TO THE SCHOOLS OF
MINES, ENGINEERING AND CHEMISTRY OF COLUMBIA
UNIVERSITY, IN THE CITY OF NEW YORK

NEW YORK
PUBLIC
LIBRARY

84
PRESS OF
THE NEW ERA PRINTING COMPANY
LANCASTER, PA.
1918



9819
Mark Patter
5-7-30
Copyright, 1918,
BY GEORGE V. WENDELL

WAS WAR
1918
WAS WAR

PREFACE

The present laboratory manual is a revision of a manual entitled Physical Laboratory Notes issued by Professor Wendell in 1913. It has been revised to meet the especial needs of students in Columbia College who are preparing for admission to the graduate schools of Mines, Engineering and Chemistry of Columbia University, and for those majoring in the natural sciences and in particular in physics and chemistry. The exercises were selected to emphasize those important fundamental principles and concepts in physics that are of vital importance to a clear understanding of the scientific and engineering courses that follow. They are, at the same time, well adapted to afford an excellent means of training the student in the handling of delicate apparatus, and in the correct methods of observing and recording data, of estimating the accuracy of the measured quantities and of preparing adequate reports,—matters of large importance in a correct experimental procedure in testing and in research laboratories. They also serve as an introduction to the leading books and tables of reference which are so indispensable in later work.

A brief discussion of the Precision of Measurements forms a necessary introduction to the experimental portion of the book, for a knowledge of the principles by which the effect of errors on the final result may be determined is recognized as necessary in the planning and conduct of any piece of investigation, whether an engineering test or a problem in research, as a means of economizing time and labor and of furnishing an estimate of the reliability of the final result.

The manual was prepared for use in the laboratory course, Physics 49-50, which consists of one three-hour period each week for the college year (two terms). The prerequisites are mathematics through the differential and integral calculus, entrance physics and a one-semester six-hour lecture and recitation course on Mechanics and Wave-Motion, and, *in parallel*, a six-hour lecture and recitation course on Heat, Light, Electricity and Magnetism.

Each exercise is complete in itself, and the student is required

to discuss the theory of the subject matter involved and to derive most of the equations. The exercises described are for the most part standard ones, the selection and arrangement of the material being based in a large measure on the experience of one of the authors in the instruction of engineering students at three of our leading schools of engineering. Many helpful suggestions have been received from various members of the instructing staff, and special thanks are due to Dr. Morris F. Weinrich of Columbia and to Professor C. B. Le Page of Stevens Institute.

G. V. W.
W. L. S.

July, 1918.

CONTENTS

	Page
LABORATORY RULES.....	1
PRECISION OF MEASUREMENTS.....	5
PROBLEMS ON PRECISION.....	37
LABORATORY EXERCISES:	

Mechanics

1. Vernier and Micrometer Calipers.....	47
2. The Spherometer.....	54
3. The Micrometer Microscope.....	57
4. Simple Harmonic Motion—Determination of g	61
5. The Beam Balance.....	65
6. Specific Gravity of Liquids—Picnometer.....	76
7. Specific Gravity of Solids—Specific Gravity Bottle	81
8. Young's Modulus by Stretching.....	83
9. Coefficient of Rigidity by Torsion.....	87
10. Torsion Pendulum—Moment of Inertia and Coeffi- cient of Rigidity.....	92
11. Radius of Gyration and Moment of Inertia of a Body by Swinging it as a Gravity Pendulum..	97
12. Moment of Inertia by Measurement of Torque and Angular Acceleration.....	100
13. Impact.....	104
14. The Mercurial Barometer.....	109
15. Boyle's Law.....	112

Light

16. Photometry.....	115
17. The Spectrometer—Index of Refraction.....	118
18. The Spectrometer—Dispersion Curve.....	124
19. The Chemical Spectroscope.....	128
20. Focal Length of Lenses and Lens Combinations...	133
21. The Compound Microscope.....	141

Heat

22. Air Thermometry.....	146
23. Coefficient of Linear Expansion.....	154
24. Pressure and Boiling Point.....	158

25. Hygrometry.....	163
26a and 26b. Specific Heat.....	168, 174
27. Mechanical Equivalent of Heat.....	182

Electricity and Magnetism

28. Construction of Voltmeter and Ammeter.....	186
29. The Wheatstone Bridge—Slide Wire and Decade Forms.....	190
30. The Potentiometer.....	195
31. Comparison of Capacities by Ballistic Galvanometer	199
32. Thermo-couple and Sensitivity of a Galvanometer	202
33. Magnetic Fields of Solenoid and Flat Coil.	204
34. Elementary Principles of the Generator.....	206
35. Calibration of Ballistic Galvanometer by Standard Solenoid and the Measurement of Magnetic Fluxes.....	208
36. Force on a Conductor Carrying a Current in Mag- netic Field.....	212

APPENDIX

Tables.....	217
Reference Books.....	223

LABORATORY INSTRUCTIONS FOR PHYSICS 49-50

Attendance

Prompt and regular attendance throughout the full laboratory period is expected of every student. No cuts are allowed. Each student is required to complete acceptably the full number of experiments. Absences due to sickness or excusable by the Dean may be made up for full credit *provided application is made to the instructor at the earliest opportunity.*

Preparation

The student must prepare for every laboratory period by studying the laboratory directions, lecture notes and references.

Precision of Measurements

Careful attention must be given to the preparation of all assignments on the precision of measurements. Talks on precision may be given in any laboratory period, and the student should be prepared at any time for quizzes on this work.

Assignment of Experiments

Laboratory assignments will either be made at the close of the laboratory period or will be posted on the bulletin board at least four days before the next period.

Observations, Data Sheets and Computations

The only paper to be used in the laboratory is size No. 6 and must not be folded. Every observation made in the laboratory is to be recorded neatly and legibly, preferably in *ink*, on the ruled side of the standard laboratory paper. On the top line of each sheet should appear the student's name and laboratory number at the right, and the experiment number at the left. Nothing else should be written above the top line and nothing in the left-hand margin.

The laboratory letters and number designating each instrument or piece of apparatus used *must* be recorded.

Records are to be made of the *actual* observations immediately when taken and are never to be the result of any mathematical calculation, however simple. No erasure on the original data sheets should ever be made. An erroneous entry should be simply crossed out with the comment, "rejected" or "mistake."

Calculations other than the averaging of a column of figures should never appear on the data sheet, but should be on the back of the preceding page. Computations are to be made by means of logarithms wherever practicable, but where its precision is sufficient the use of a slide rule is recommended. It is desirable that the substitution of the measured quantities in the formulas used should be clearly indicated in the report. This may appear on the data sheet so placed as to be entirely distinct from the data.

When the data are complete they should be presented at once to the instructor for his indorsement. This indorsement does *not* necessarily indicate that the results are sufficiently accurate to be acceptable.

Too much care cannot be taken in the *preparation* of the *original data sheets* in all experimental work, such as engineering tests and research work. The data should be neatly and clearly recorded, preferably on one side of the sheet only, every item so carefully labeled that the data will be intelligible after a lapse of time when the details of the test may be forgotten. The data sheet should contain the date on which the observations were made, the identification numbers of all instruments, and a record of any important conditions that may have influenced the results, and should bear the signatures of the observers. The idea is to make the original data sheets so complete and clear that the records may be understood by anyone who has occasion to consult them and may be offered as evidence if the occasion arises. To do this it will be necessary to plan the data sheet before taking any observations. It is advisable to present the plan to the instructor for criticism.

Reports

Reports must be arranged in the following order:

- (a) A concise statement of the object of the experiment.
- (b) A brief analysis of the entire problem. This should give a concise review of the whole experiment, a description of the apparatus, the precision sought, the physical principles involved, the fundamental definitions and the method chosen. It should

never be a copy of the discussion in the manual but should serve as a means of developing originality of thought and expression. Diagrams are usually an assistance in this discussion and are imperative in all exercises on electricity. For electrical symbols consult the chart posted in the laboratory.

(c) A brief summary of the results obtained, a discussion of their reliability, and a statement of the physical conclusions arrived at.

(d) The data sheets arranged in order. In general the computations corresponding to each page of data should be on the left hand page opposite. Throughout the report nothing should appear on the left hand page except computations. All final results should be on the right hand pages and should be properly labelled. The original data sheets bearing the instructor's indorsement must always be included, even though a copy of the data is also incorporated.

(e) All curves, plotted on the standard cross-section paper, should follow at the end of the report. (See notes on curve plotting, page 32.)

Reports are to be bound in the standard covers using only "No. 2 fasteners."

Grading and Delivery of Reports

Reports are due in the box provided before 12:30 of the day *preceding* the next laboratory period. Any report returned for correction is due at the time specified (by stamp or in writing) by the instructor. Should no time be specified, the report is due not later than 12:30 of the day preceding the following laboratory period. When the corrected report is returned by the student, it must contain both the *original* and the *corrected* data and computations. No marks of correction should be erased unless authorized by the instructor.

A deduction of 3 points will be made on all *late* reports. More will be deducted from the grade for each additional day the report is late. *Reports one week late will receive zero credit.* In case of unavoidable lateness application should be made at once (by mail if necessary) to the instructor for an extension of time. Such extensions are given only in writing. No allowance will be made for lost data sheets or reports.

The grade given on any experiment will be affected by the

neatness and arrangement of the data, accuracy of the data, correctness of computations and precision of results, the value of the discussion, and the solution of any problems assigned.

The condition in which the apparatus is left at the close of the laboratory period will also be taken into account.

The final grade for the course will be based on the following:

1. Grades on the individual experiments.
2. Written or informal oral quizzes on the laboratory work and on precision of measurements during each semester.
3. Written examinations at the end of each semester.

If the grade in any one of the component parts is deficient, the whole course will be considered as incomplete.

Additional Directions

Each student is held responsible for any directions which may be posted on the bulletin board.

PRECISION OF MEASUREMENTS

References to larger works on the Theory of Errors and Precision of Measurements:

Wright and Hayford—Adjustment of Observations.

Holman—Discussion of the Precision of Measurements.

Holman—Computation Rules and Logarithms.

Weld—Theory of Errors and Least Squares.

Palmer—Theory of Measurements.

Goodwin—Precision of Measurements and Graphical Methods.

The following notes on the Precision of Measurements are intended to give the student an insight into methods of determining the reliability of experimental results and the proper use of significant figures in computing and expressing such results. They are not intended to be an exhaustive treatise on the Theory of Errors, but they contain the results of the theory as well as the results of years of laboratory experience which have been found helpful in planning and analyzing experimental work. The emphasis is laid on the regular work of a physics laboratory, but the principles are generally applicable to scientific experimentation.

Importance of the Subject.—It is of very great importance in all experimental work to be able to examine the sources of error intelligently and then to estimate the reliability of the result. This is of course absolutely necessary in the determination of physical constants and in calibration work in general. Its importance in engineering depends on the particular piece of work in hand. An allied problem is perhaps of even greater interest to the engineer, namely, what grade of apparatus is required to obtain a result within certain desired limits. Will a ten-dollar instrument do the work, or is a hundred-dollar instrument required? Can the experimental portion of the work be intrusted to unskilled labor, or does it require the services of expert workmen or technically trained men? Though the solution of many of these questions must rest upon experience, the thoughtful application of the principles of the Precision of Measurements will be found of great assistance. Furthermore, these principles

will make it possible to secure the desired reliability in the result without unnecessary labor or the waste of much valuable time in experimentation and computations.

Mistakes.—Carelessness on the part of the observer is a common source of serious inaccuracies in experimental work, especially in the case of beginners. These inaccuracies cannot be called errors in the technical sense of the word, nor can the theory of errors be applied to them. They are really blunders or mistakes,—mistakes in arithmetic, in the recording of observations, in reading the wrong number on the scale, in making an incorrect reading through carelessness in noting the value of the smallest scale division, in assuming a pyrometer to read centigrade temperature when it reads Fahrenheit, and so on. Individual vigilance and the habit of doing everything in a careful and orderly manner are the only means of reducing such inaccuracies to a minimum. It is often highly advisable to run some rough independent check experiment or to test the final results with common sense to see that no gross blunder has been committed. For example, in making an accurate measurement of a length with a dividing engine, it is advisable to measure the length with an ordinary centimeter scale as a rough check.

Errors due to Conditions.—It often happens that the physical conditions under which the experiment is being performed are at variance with the requirements. The relative humidity may be too high; or the wind may be blowing when the apparatus requires a quiet atmosphere; or the sun may shine on a part of the apparatus, thus producing disturbing temperature differences; or street traffic may produce vibrations where freedom from jar is necessary. Better conditions must be sought before good work can be done.

Errors in Instruments.—The science of instrument making has advanced to such a degree that in most of the ordinary measuring instruments such errors as faulty graduation of scales, eccentricity of circles, defect in a screw, inequality in length of balance arms, have been reduced to a minimum. Their effects cannot, however, be overlooked in accurate work. Before an instrument can be used with precision, errors of this nature, to which it is subject, must be carefully studied and understood. Whenever possible, it is customary to overcome such defects by

the direct calibration of the instrument against a carefully calibrated standard. Such comparisons may be made by the observer if in possession of a standard instrument, or the instrument may be sent to the Bureau of Standards in Washington, where a careful calibration will be made by government officials and an official certificate of corrections issued with their reliability. Instruments that are liable to alter their adjustment with wear or age, such as watches, thermometers, standard lamps, electrical apparatus, balance weights, aneroid barometers, etc., should be rechecked now and then against secondary standards to determine whether their calibration has altered.

It not infrequently happens that there is no standard instrument for the comparison. The apparatus must be standardized by original experimentation and research. That is the type of problem which the workers at a Bureau of Standards must face. Such work is however almost always dependent on the use of one or more instruments already standardized,—some fundamental instrument for the measurement of mass, length or time.

After an instrument is calibrated, it must be properly used or errors will be introduced. A metal scale, such as the scale on a barometer, may be calibrated at one temperature but used at another, or a resistance box calibrated at 20° C. may be used for the measurement of resistance at some entirely different temperature. In each case repeated measurements may agree very closely among themselves, and yet the result will be in error. If the law of expansion of the metal with temperature or of resistance change with temperature is known, the error thus introduced may be found and the proper correction applied. To be sure, the experimental determination of the coefficient of linear expansion or of the temperature coefficient of resistance is not perfect, but the errors remaining, sometimes called *residual errors*, are usually quite small. Limiting values may be assigned to them, and their influence on the accuracy of the result may be estimated.

Systematic Errors.—When the results of a set of data are examined, they often show a drift of error in one direction, that is, the errors all have the same algebraic sign and can be shown to bear a fixed relation to some condition. For example: the angular twists per 200 grams load on the torsion head of a rod were found for the various lengths as follows:

Length.	Twist.
100 cm.	14.32°
50 "	7.35°
25 "	3.87°
12.5 "	2.07°

The ratios of successive twists should have been constant but were found to be

0.513, 0.526, 0.537,

indicating an increasing ratio. The rod was examined carefully and found to have a constriction near the torsion-head end which played a larger and larger part as the lengths were decreased. The source of error was thus located and the condition corrected.

Errors due to Prejudice and Personal Equation.—The observer should be in normal physical condition, a fatigued observer being seldom a good one. He should also be *unprejudiced*, not trying to obtain a certain reading or straining a point to verify some preconceived theory. The beginner is usually prejudiced by his first observation. For some unknown reason he seems to consider the first reading the best and seeks to make subsequent readings conform as nearly as possible to that one.

Unprejudiced, skilled observers differ among themselves in judgment. This is quite apparent in experiments on photometry or color matching, and also in timing results. The time of reaction is different in different observers. This gives rise to what is known as *personal errors* or the *personal equation*. The best that can be done in such cases is to take the average result from a number of observers. One of the objects in the refinement of instruments is to reduce this personal equation to a minimum, or, in other words, to make observations as impersonal as possible.

Accidental Errors.—In addition to the sources of error already enumerated, there still remain errors that are entirely beyond the control of the observer, whose cause is temporary, lasting only during a single observation and disappearing upon a slight change of conditions. The cause may be external or it may be psychological. For example, the individual observations on a measured quantity may differ through sudden fluctuations in temperature, sudden barometric changes, the jarring of the apparatus by passing vehicles or the wind, variations in the

earth's magnetism or in the amount of magnetic material present in magnetic experiments, or through inattention, or fatigue of eye or ear. The presence of such errors is made evident by the discrepancies between individual observations upon the same quantity made *apparently* under identical conditions, with the same care and by the same observer, and when all known sources of disturbance have been removed. Such errors are called *accidental errors*, and they can be greatly reduced if not wholly eliminated by increasing the number of observations.

Experience has demonstrated that when the errors are purely accidental, the *arithmetical mean* or average of a group of observations made with equal care and meriting the same confidence has a larger probability of being the true value than has any single observation. It has also shown that the reliability of the arithmetical mean increases as the square root of the number of observations. This would mean that for an infinite number of observations the error of the mean would be zero as far as the accidental errors are concerned. It would of course require an infinite time to take the observations and to average them. Consequently the number should be determined by the precision sought and the errors present due to other sources.

If n observed readings $a_1, a_2, a_3, \dots, a_n$ of a quantity are made with equal care and skill, then the arithmetical mean m is:

$$m = \frac{a_1 + a_2 + \dots + a_n}{n}$$

The difference between any single observation and the mean is called a *deviation* of that observation from the mean, or

$$d_1 = a_1 - m, \quad d_2 = a_2 - m, \quad \dots, \quad d_n = a_n - m.$$

A deviation, also called a residual, should never be confused with the *absolute error* of the observation which, strictly speaking, is the difference between the *observed* value and the *true* value of the measured quantity.

The algebraic sum of the deviations or residuals of such a series of observations from the mean is zero, and the sum of the squares of the deviations is a minimum, *i. e.*, is less than would have been obtained from any number other than the arithmetical mean. This is in accordance with the adjustment of observa-

tions by the Principle of Least Squares, which states that "the most probable value of a measured quantity that can be deduced from a series of direct observations, made with equal care and skill, is that for which the sum of the squares of the deviations is a minimum."

Experiment has shown that in the case of *accidental errors* when the number of observations is large, (1) deviations of small magnitude occur more frequently than large ones, (2) positive and negative deviations of the same magnitude are equally likely to occur, (3) very large deviations are very unlikely to occur. These facts can be expressed mathematically by the law of chance, namely,

$$y = \frac{h}{\sqrt{\pi}} e^{-hx^2}$$

where y is the frequency of occurrence of a deviation of magnitude x , e is the logarithmic base (2.71828) and h is an abstract number sometimes taken as a numerical estimate of the reliability of the observations and called the precision index. For the derivation of this equation reference should be made to works on the theory of errors and least squares. It can be shown that the constant h is given by

$$h = \frac{\sqrt{n(n-1)}}{\sqrt{\pi}\Sigma(d)}$$

where n is the number of observations and $\Sigma(d)$ is the sum of the deviations.

Deviation Measures.—Since the true value of a measured quantity is generally unknown, the actual error of each observation and of the mean cannot be determined. The deviations or residuals $d_1, d_2, d_3, \dots, d_n$ give the amounts by which the individual measurements differ from the arithmetical mean. If the average of these deviations be found, disregarding the signs, the value thus obtained, called the *average deviation* of a *single* observation from the mean,

$$\text{a.d.} = \frac{d_1 + d_2 + \dots + d_n}{n}$$

represents how much on the average any single observation of a series will differ from the arithmetical mean as far as accidental

errors affecting the measurement are concerned. Other methods used by experimenters for estimating the probability that the error of a single observation will lie between certain limits are (1) the mean error μ which equals the square root of the mean square (root mean square), or

$$\mu = \sqrt{\frac{\Sigma(d^2)}{n}}$$

and (2) the probable error ϵ .

The term "probable error" of an observation is misleading since ϵ is not the most probable error, but is an error of such magnitude that any given observation is just as likely to deviate from the mean by an amount numerically greater than ϵ as it is to deviate by less than ϵ . If the number of readings is large and the nature of the deviations purely accidental, then it is shown that out of 1,000 deviations

54	are less than	0.1 ϵ
264	" " "	0.5 ϵ
500	" " "	1.0 ϵ
823	" " "	2.0 ϵ
957	" " "	3.0 ϵ
999	" " "	5.0 ϵ

The following table gives the numerical relation between these three terms for a long series of observations of equal reliability.

	μ	ϵ	a.d.
$\mu =$	1.0000	1.4826	1.2533
$\epsilon =$	0.6745	1.0000	0.8453
a.d. =	0.7979	1.1829	1.0000

For example, the probable error $\epsilon = 0.85$ a.d. (approx.).

Deviation of the Mean, A.D.—What is particularly desired in experimental work is an estimate of the reliability of the arithmetical mean in *so far as accidental errors are concerned*. The previous deviation measures can hardly be used since they do not take into account the number of observations, and it is known from the theory of probability that an arithmetical mean computed from n equally trustworthy observations is more reliable than any one observation of the series as $\sqrt{n} : 1$. Con-

sequently in this course the measure of the reliability of the mean will be given by the quantity A.D., where

$$\text{A.D.} = \frac{a.d.}{\sqrt{n}}$$

The reliability of the mean may also be expressed as a percentage deviation $100 \text{ A.D.}/m$, where m is the arithmetical mean.

Typical Example.—The terms thus far discussed under the head of accidental errors can probably be made clearer by a practical illustration, as follows: The voltage of the local continuous current generator circuit varies from instant to instant. The fluctuations are beyond the control of the observer and may be considered to follow more or less truly the theoretical “accidental” conditions. To obtain numerical data a lamp was connected in the circuit and the voltage between its terminals recorded by a voltmeter, reading directly to volts, every fifteen seconds until 220 observations had been made. The arithmetical mean of the 220 readings was 117.557 volts, the extreme values being 122.5 volts and 114.0 volts. The sum of the deviations, neglecting algebraic signs, was 204.6 volts = $\Sigma(d)$, and the average deviation of a single observation from the mean was $a.d. = 205/220 = \pm 0.93$ volt. The probable error ϵ was found by the relation $\epsilon = 0.85 (a.d.) = \pm 0.79$ volt. The precision index h was computed by the formula given above and was found equal to 0.606. The theoretical smooth curve of error for this value of h (see Fig. A) was obtained by substituting this value in the equation of error

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}$$

It might be noted here that the smaller the deviations the larger is the value of h in the curve of error.

The irregular curve was obtained by plotting the number of observations having deviations from the mean within a given interval against the magnitude of the deviation. The size of the interval chosen was 0.75 volt. For example, 21 positive deviations lay between +0.75 and +1.5 volt.

An examination of the observations showed 106 readings larger than the mean and 114 less. Of the + deviations 42 were larger than 0.79 volt and 64 were less than 0.79 volt. Of the

— deviations 52 were larger than 0.79 volt and 62 were less than 0.79 volt. Only one reading exceeded the arithmetical mean by as much as 4.9 volts, and only one was less than the mean by 3.6 volts. These figures will give some idea of the agreement to be expected between theory and practice. The larger the number of observations the closer will be the agreement.

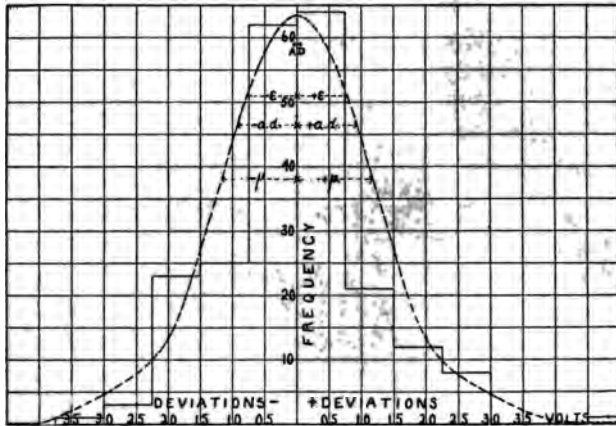


FIG. A.

In the voltmeter illustration the mean of twenty observations (40th to 60th) was 117.04 volts with an a.d. = ± 0.83 volt. The mean of the 220 readings was 117.56 with an a.d. = ± 0.93 volt. The mean of the 220 observations is certainly more representative of the average voltage in point of time than the mean of the 20 readings, even though the a.d. of the latter is smaller than the a.d. of the 220 readings. The greater reliability of the mean of 220 readings will be evident if the A.D.'s be computed.

$$A.D._{220} = \frac{\pm 0.93}{\sqrt{220}} = \pm 0.063 \text{ volt.}$$

$$A.D._{20} = \frac{\pm 0.83}{\sqrt{20}} = \pm 0.19 \text{ volt.}$$

Best Value for n .—Since the reliability of the arithmetical mean of a series of observations was found to increase as the

square root of the number of observations n , the error due to accidental causes can be practically eliminated if n is made sufficiently large. However, the labor involved varies directly with n while the reliability increases only as the $\sqrt{n}$. Consequently a point is quickly reached beyond which the small gain in the reliability through the increase in the number of observations is not justified. For example, the reliabilities of sets of 25, 16, 9, 4 observations are to one another only as 5: 4: 3: 2 respectively. In ordinary laboratory practice four or five careful observations usually suffice. This number must be determined by the variations among the readings, the precision sought and the errors due to other sources. If the accidental errors are large, the number of observations must be correspondingly large.

Deviation Measure vs. Precision Measure.—The deviation measure A.D. furnishes a method of estimating the reliability of the arithmetical mean of a series of observations in so far as *accidental errors* are concerned. Were no other errors present, this would serve as the *precision* measure, p.m., of the result, by which is meant the best numerical measure or estimate of the reliability of the result after all known sources of error have been eliminated or corrected. In general there are other sources of error present, such as constant instrumental errors and errors of method. For example, a scale may be used at a temperature other than that at which it was calibrated, or in the use of a sensitive centigrade thermometer, the zero and hundred points may not represent the exact readings in ice and steam respectively. Though the arithmetical mean may be corrected for such errors, the corrections cannot be exact and hence the final result will be in doubt on account of these *residual errors*. The magnitude and sign of these residual errors $e_1, e_2, \dots, e_n$ cannot be determined, but it is often possible to assign an upper limit to their value and to treat them in accordance with the law of chance. If they are not negligible in comparison with the deviation measure A.D., the most probable combined error E due to the various causes may, *provided the errors are independent of one another*, be found by taking the square root of the sum of their squares, or

$$E = \sqrt{(\text{A.D.})^2 + e_1^2 + e_2^2 + \dots + e_n^2}$$

and the value E may be taken as the precision measure, p.m., of

the final result. All the quantities in this expression must be in the same units, either in units of the measured quantity or in percentage or fractional deviations. This value of E is a more probable estimate of the reliability than the sum of the errors, since all the errors are stated as being plus or minus with equal probability, and, consequently, with a chance that some of the errors will neutralize others.

Problem.—In the voltmeter problem on page 12, the arithmetical mean was found to be 117.56 volts with an A.D. = ± 0.06 volt. On calibrating the voltmeter it was found to read too high at this point on the scale by 0.85 volt, this calibration correction being given to ± 0.10 volt. The most plausible value of the voltage is, therefore,

$$117.56 - 0.85 = 116.71 \text{ volts}$$

with a precision measure,

$$\text{p.m.} = \sqrt{(.06)^2 + (.10)^2} = \pm 0.12 \text{ volt}$$

instead of 117.56 volts ± 0.06 volt.

Expressing the precision measure in per cent. gives

$$\text{p.m. in per cent.} = \frac{\pm 0.12}{117} \times 100 = \pm 0.10 \text{ per cent.}$$

and the result should be stated as 116.71 volts within ± 0.10 per cent. In most cases it is preferable to express the precision measure in per cent. since the latter is independent of the units used. For example, the above value could be expressed as 116,710 ± 120 millivolts, but the percentage precision would still be ± 0.10 per cent.

The percentage precision might have been computed directly, thus:

$$\text{p.m. in per cent.} = \sqrt{\left(\frac{0.06}{117} \times 100\right)^2 + \left(\frac{0.10}{117} \times 100\right)^2} = \pm 0.10 \text{ per cent.}$$

Precision Measure vs. Accuracy.—When the true value is known the “accuracy” may be expressed as the difference between the experimental quantity obtained and this true value. Since, however, the exact or true value is seldom known, the accuracy of the result cannot be stated, and it becomes the more imperative to have methods of estimating the precision measure or reliability of the result of a series of observations.

The student is also warned against confusing *sensitivity* with *precision measure* and with *accuracy*. An instrument may be twice as sensitive as it is accurate or vice versa. It is interesting to note that by taking a large number of readings the average or arithmetical mean may have meaning beyond the sensitivity

of the instrument on which the readings are taken. Thus in the voltmeter problem, the individual readings had to be estimated in the tenths place of volts, but the result is significant to hundredths of a volt as far as the accidental errors affect it.

Rejection of Observations.—When a single observation differs widely from the others, one is tempted to reject it. This must be done with great care, especially when the number of observations is small, as is usually the case. It is often very difficult to decide whether a deviation is in the regular course of accidental errors or whether it is due to a blunder or mistake. Various authorities recommend various tests to be applied for rejections. From an inspection of the probability data given on page 11, it would appear to be reasonably safe to reject an observation if it differs from the mean by as much as four times the average deviation, the mean being computed first with the doubtful reading omitted. The reading at all events should not be taken from the data sheet but should be marked "Reject" and left for the inspection of others who may be interested.

Definition of a Significant Figure.—In this course the precision measure of a result will be the criterion for determining how many of the figures are significant. For example, the mass of an object is obtained from the following set of weighings:

Weighings.	Deviations.
20.7063g.	— 0.00001g.
.7060	— 31
.7060	— 31
.7049	— 141
.7069	+ 59
.7078	+ 149
.7062	— 11
.7063	— 1
.7064	+ 9
9 568	9 433
20.70631g.	± 0.00048g. = a.d.

$$\text{A.D.} = \frac{0.00048}{\sqrt{9}} = \pm 0.00016g.$$

If the constant and systematic errors are negligible, then ± 0.00016 becomes the precision measure and the mass is

$$20.70631 \pm 0.00016 \text{ grams.}$$

That is to say, the 3 is in doubt by 1.6 and the 1 is in doubt by 16. *We shall consider a figure as experimentally significant until it is in doubt by 80 or more.* This number therefore has seven significant figures. Had the A.D. come out to be 0.00086, the 1 would have been dropped and the number expressed as 20.7063 ± 0.0009 grams, there being no point in expressing the A.D. or precision measure itself beyond the last place of significant figures. *The A.D. will itself be expressed with only two significant figures until the first one is as large as 8, after which only one is to be used.*

This number expressed in kilograms is 0.02070631 kg. but it still has only seven significant figures, the first two zeros being used merely to locate the decimal point. If the last figure comes out zero it should be recorded the same as any other digit, showing that it is a measured quantity and in doubt by something less than 80. It is then a significant zero. For example, 4730.0 ± 1.2 grams.

In rejecting figures beyond the last place of significant figures, it is well to raise the last place retained by 1 if the next figure, which is to be dropped, is 5 or greater. For example: 5.3197 ± 0.0655 becomes 5.320 ± 0.066 with 4 significant figures while 60.54 good to 4.0 parts in 100 becomes 60.5 ± 2.4 with 3 significant figures. The number of significant figures in a quantity like 76,300,000 cannot be told unless the precision measure be stated. If the precision measure is given as $\pm 57,000$ the number has 5 significant figures and should be expressed 7.6300×10^7 .

Never express a quantity with more figures than called for by this rule, for if the rule errs, it errs on the side of conservatism. It would be better to use fewer figures rather than more.

For computing residuals or deviations the mean should be carried out one place further than the measured quantities. The A.D. will tell whether the last place in the mean is to be retained or dropped in the final statement of the result.

Rules for Computation.—An experimental quantity is something different from an ordinary arithmetical number. In arithmetical numbers as such there is no element of uncertainty and hence in their sums, differences, products and quotients there can be no uncertainty. For example, $\frac{1}{3}$ of 100 is $33.3333\frac{1}{3}$, but $\frac{1}{3}$ of 100.0 ± 2.6 grams is 33.3 ± 0.9 grams. More figures would have no meaning. Much time and effort are often wasted in obtaining experimental data and in making computations by a

failure to recognize this distinction, and the chances of error are increased by using more figures than are necessary. Moreover incongruities in the use of significant figures arouse skepticism as to the competence of the experimenter in other directions.

The following rules will serve as a guide to intelligent computation.

RULE I.—In addition and subtraction the result cannot be reliable beyond the last significant figure in any one of the terms. Hence, find the precision measure of each of the quantities (expressed in units of the quantity itself, not in per cent.) and retain figures in all the quantities only as far as called for by the quantity with the largest precision measure.

Examples.—

1. To add

Number.	A. D.	Computation.
2807.5	± 2.8	2807.5
0.0648	± 0.0015	0.1
83.8954	± 0.0046	83.7
625.0	± 1.2	625.0
		3516.3 Ans.
		* *

Using all the figures in the data the sum is 3516.2602, a number that differs from 3516.3 by less than one unit in the second unreliable place of figures. The unreliable places of figures are starred.

2. To subtract

Number.	A. D.	Computation.
99.85	± 0.30	99.85
0.0373	± 0.0018	0.04
		99.81 Ans.
		* *

RULE II.—In multiplication and division the percentage precision must be the guide. If any term in such an operation is in doubt by say 5 per cent., the product or quotient will be in doubt by 5 per cent. on account of that term alone. The product or quotient cannot therefore be more precise than the *percentage precision* of the *least* precise factor entering into the computation. If this is

1 per cent. or more, retain four significant figures throughout the computation.

0.1 per cent. or more, retain five significant figures throughout the computation.

0.01 per cent. or more, retain six significant figures throughout the computation.

Example.—Solve $\frac{2807.5 \times 83.6594}{0.0646}$, given the following data:

2807.5 ± 2.8 or reliable to ± 0.10 per cent.

83.6594 ± 0.0046 or reliable to ± 0.0055 per cent.

0.0646 ± 0.0015 or reliable to ± 2.3 per cent.

Carry four figures in the computation as determined by the last factor, thus:

$$\begin{array}{r}
 83.66 \\
 2808 \\
 \hline
 66928 \\
 66928 \\
 16732 \\
 \hline
 0.0646 \overline{) 234900.} \quad 3.636 \times 10^6 \text{ Ans.} \\
 \underline{1938} \\
 4110 \\
 \underline{3876} \\
 2340 \\
 \underline{1938} \\
 4020 \\
 \underline{3876}
 \end{array}$$

RULE III.—In using logarithms for computing products and quotients, retain as many figures in the *mantissa* as are retained in the factors themselves under Rule II. Thus to solve the above problem by logarithms:

$$\log 83.66 = 1.9225$$

$$\log 2808 = 3.4484$$

$$5.3709$$

$$\log 0.0646 = 2.8102$$

$$\text{illog } 6.5607 = 3.637 \times 10^6$$

General Remarks.—In applying Rules II and III to cases involving *roots*, it may not be necessary but it is *safer* to multiply the precision of the quantity by the *root* before applying the test.

Thus in computing $X = \frac{pq}{r^{1/3}}$

with p reliable to 0.23 per cent.

q reliable to 0.52 per cent.

r reliable to 1.5 per cent.

use five figures throughout the computation since $r^{1/3}$ will be reliable to $\frac{1}{3}$ of 1.5 = 0.5 per cent. See page 23, Note II.

The rules given above are stringent enough to guard against any serious accumulation of error made by sixteen rejections of superfluous figures. One seldom has occasion to use as many operations as this, so that the rules might be somewhat relaxed where only three or four rejections occur. Never use more figures than are called for by these rules.

The Slide Rule.—The use of the slide rule is strongly recommended to save time and labor. It should be remembered, however, that it cannot be relied upon in computations calling for more than four significant figures.

Caution.—One must not be misled to think that Rules I, II and III give the number of significant figures in the final result. These rules are for computations only. The number of significant figures in the final result will be determined by the precision measure of the result. How this is to be obtained in indirect measurements is discussed in the following pages. It need only be noted here that the final result will *not* have *more* significant figures than the rules call for. It may have fewer.

A more extended discussion of *significant figures* and *computations* can be found in Holman's Precision of Measurements and in his Computation Rules and Logarithms.

Precision of Indirect Measurements

The Direct Problem.—On page 14 it was shown how the precision measure of a *directly measured quantity* can be obtained by finding the independent errors produced in it by various sources and then taking the square root of the sum of the squares of these independent errors. In most experimental work, however, the numerical value of a desired quantity cannot be measured directly but it can be expressed as a function of one or several quantities that are capable of direct measurement. For example, the acceleration of gravity g cannot be measured directly. Its value can be found by the measurement of the length l and the period t of a pendulum and the use of the relation

$$g = \frac{4\pi^2 l}{t^2}.$$

The determination of l and t may in turn require several operations. Likewise, to obtain the brake horse power of an engine

$$\text{h.p.} = \frac{2\pi FhN}{33000}$$

requires the measurement of a force, of a lever arm and of revolutions per minute. The solution of the precision measure of such *indirect measurements* requires special attention.

Let the quantity sought Q be a function of the measured quantities $q_1, q_2, q_3, \dots, q_n$, i. e., $Q = f(q_1, q_2, q_3, \dots, q_n)$. It is desired to find the reliability of Q having given the precision measures, $\Delta q_1, \Delta q_2, \Delta q_3, \dots, \Delta q_n$ of the measured quantities. The process consists in finding the errors $e_1, e_2, e_3, \dots, e_n$ produced separately in Q by the errors $\Delta q_1, \Delta q_2, \dots, \Delta q_n$ and then taking the square root of the sum of the squares of these errors provided they are entirely *independent* of one another and their *algebraic sign is unknown*, or calling E_Q the total variation in Q ,

$$E_Q = \sqrt{e_1^2 + e_2^2 + e_3^2 + \dots + e_n^2}.$$

In this expression

$$e_1 = \frac{\partial Q}{\partial q_1} (\Delta q_1)$$

since $\partial Q/\partial q_1$ (the partial derivative of Q with respect to q_1) represents the variation in Q per unit variation in q_1 , the remaining q 's being assumed constant, and, therefore, a deviation Δq_1 in q_1 must cause a deviation of $\partial Q/\partial q_1 (\Delta q_1)$ in Q . Similarly,

$$e_2 = \frac{\partial Q}{\partial q_2} (\Delta q_2), \dots, e_n = \frac{\partial Q}{\partial q_n} (\Delta q_n).$$

It is important to note that each term of the form $\partial Q/\partial q_1 (\Delta q_1)$ has the dimensions of Q and not of q .

To illustrate the above principles take the following simple problem. Suppose the total cost of travel between New York and Chicago is $\$Q$. This is due to q_1 miles traveled at a railroad rate of $\$0.025$ per mile, q_2 hours in the Pullman at a rate of $\$0.20$ per hour, and q_3 meals at a rate of $\$1.25$ per meal. The function is

$$\$Q = q_1 \times \frac{\$0.025}{\text{mile}} + q_2 \times \frac{\$0.20}{\text{hour}} + q_3 \times \frac{\$1.25}{\text{meal}}.$$

What will be the effect on Q of two hours more or less in the Pullman?

$$\frac{\partial Q}{\partial q_2} (\Delta q_2) = \frac{\$0.20}{\text{hour}} \times (\pm 2 \text{ hours}) = \pm \$0.40.$$

The result comes out in dollars and not in miles, or hours, or meals, or per cent.

If $q_1 = 1,000$ miles, $q_2 = 12$ hours, $q_3 = 3$ meals, then $Q = \$31.15$ and the percentage effect on Q of ± 2 hours in the Pullman is

$$\pm \frac{\$0.40}{\$31.15} \times 100 = \pm 1.28 \text{ per cent.}$$

Whereas the percentage variation in q_2 is

$$\pm \frac{2}{12} \times 100 = \pm 16 \text{ per cent.}$$

Typical Problems:

1. The dimensions of a block are given as follows: $l = 10.000 \pm 0.030$ cm.; $l = 20.000 \pm 0.040$ cm.; $b = 5.000 \pm 0.025$ cm. The length of a line around the block is $L = 2l + 2b$. What is the actual and the percentage precision in L ?

$$\begin{aligned} \text{The actual precision } E_L &= \sqrt{\left[\frac{\partial L}{\partial l} (\Delta l) \right]^2 + \left[\frac{\partial L}{\partial b} (\Delta b) \right]^2} \\ &= \sqrt{(0.060)^2 + (0.050)^2} = \pm 0.09 \text{ cm.} \end{aligned}$$

$$\text{The percentage precision} = \frac{\pm 0.09}{50} \times 100 = \pm 0.18 \text{ per cent.}$$

The length is $L = 50.00 \pm 0.09$ cm.

Note.—The deviation in l of ± 0.04 cm. causes a deviation in $2l$ of ± 0.08 cm. with a consequent deviation of ± 0.08 cm. in the length L .

2. Find the actual and the percentage precision of the surface of the block.

$$\begin{aligned} A &= 2hl + 2hb + 2lb \\ \frac{\partial A}{\partial h} (\Delta h) &= (2l + 2b)(\Delta h) = 50 \times 0.030 = \pm 1.5 \text{ cm}^2 \\ \frac{\partial A}{\partial l} (\Delta l) &= (2h + 2b)(\Delta l) = 30 \times 0.040 = \pm 1.2 \text{ cm}^2 \\ \frac{\partial A}{\partial b} (\Delta b) &= (2h + 2l)(\Delta b) = 60 \times 0.025 = \pm 1.5 \text{ cm}^2 \end{aligned}$$

Then

$$E_A = \sqrt{(1.5)^2 + (1.2)^2 + (1.5)^2} = \pm 2.4 \text{ cm}^2$$

$$\text{The percentage precision} = \frac{\pm 2.4}{700} \times 100 = 0.34 \text{ per cent.}$$

The area becomes $A = 700.0 \pm 2.4$ cm².

3. Find the actual and the percentage precision of the volume of the block

$$\begin{aligned} V &= hlb \\ \frac{\partial V}{\partial h} (\Delta h) &= lb(\Delta h) = 100 \times 0.030 = \pm 3.0 \text{ cm}^3 \\ \frac{\partial V}{\partial l} (\Delta l) &= hb(\Delta l) = 50 \times 0.040 = \pm 2.0 \text{ cm}^3 \\ \frac{\partial V}{\partial b} (\Delta b) &= lh(\Delta b) = 200 \times 0.025 = \pm 5.0 \text{ cm}^3 \end{aligned}$$

Then

$$E_V = \sqrt{(3.0)^2 + (2.0)^2 + (5.0)^2} = \pm 6.2 \text{ cm}^3$$

The percentage precision = $\frac{6.2}{1000} \times 100 = \pm 0.62$ per cent. The volume becomes $V = 1000.0 \pm 6.2 \text{ cm}^3$.

NOTE. I. When the function is the product or quotient of a series of factors entering *only* to the *first power*, *e. g.*,

$$Q = \frac{q_1 q_2 q_3 \cdots}{q_7 q_8 q_9 \cdots},$$

considerable time and labor will be saved by using the percentage deviations rather than the actual deviations in estimating the percentage or the actual deviation in Q . This results from the fact that in such a function the percentage error in any component factor produces the same percentage error in the final quantity, that is

$$100 \frac{\Delta q_n}{q_n} = 100 \frac{\frac{\partial Q}{\partial q_n} (\Delta q_n)}{Q}.$$

Consequently, if E is the actual error in Q caused by errors $e_1, e_2, e_3, \dots, e_n$ which arise from errors in the components (see page 21), the percentage error in Q will be given by

$$100 \frac{E}{Q} = \sqrt{\left(100 \frac{\Delta q_1}{q_1}\right)^2 + \left(100 \frac{\Delta q_2}{q_2}\right)^2 + \cdots + \left(100 \frac{\Delta q_n}{q_n}\right)^2}$$

from which the actual error E can be readily computed.

Using the data of problem 3, the percentage precision of Q (in this case a volume) becomes

$$\begin{aligned} 100 \frac{E}{Q} &= \sqrt{\left(100 \frac{0.030}{10}\right)^2 + \left(100 \frac{0.040}{20}\right)^2 + \left(100 \frac{0.025}{5}\right)^2} \\ &= \pm 0.62 \text{ per cent.} \end{aligned}$$

The actual error E is then $(1000)(0.0062) = 6.2 \text{ cm}^3$.

NOTE II. When the function is the product or quotient of a series of factors involving powers or roots, such as

$$Q = \frac{q_1^n \times q_2^p}{q_3^r},$$

it should be remembered that a percentage deviation in q_1

produces a percentage deviation n times as great in Q since

$$\frac{\frac{\partial Q}{\partial q_1} \Delta q_1}{Q} = \frac{n q_1^{n-1} \times q_2^p}{q_3^s} \Delta q_1 = n \frac{\Delta q_1}{q_1},$$

and similarly for the other factors. Consequently the percentage precision in Q becomes

$$100 \frac{E}{Q} = \sqrt{\left(100 n \frac{\Delta q_1}{q_1}\right)^2 + \left(100 p \frac{\Delta q_2}{q_2}\right)^2 + \left(100 s \frac{\Delta q_3}{q_3}\right)^2}.$$

For example, let

$$Q = \frac{q_1^2 \times q_2^3}{q_3^{1/4}}.$$

The error e_1 in Q due to an error Δq_1 in q_1 is

$$e_1 = \frac{\partial Q}{\partial q_1} (\Delta q_1) = \frac{2q_1 q_2^3}{q_3^{1/4}} (\Delta q_1)$$

and the percentage effect is

$$100 \frac{e_1}{Q} = 100 \frac{\frac{2q_1 q_2^3}{q_3^{1/4}} (\Delta q_1)}{\frac{q_1^2 q_2^3}{q_3^{1/4}}} = 2 \left(100 \frac{\Delta q_1}{q_1}\right).$$

Similarly a ± 0.80 per cent. error in q_2 will produce a ± 2.4 per cent. error in Q , while a ± 1.6 per cent. error in q_3 will produce a ± 0.40 per cent. error in Q .

4. The index of refraction is given by the equation

$$\mu = \frac{\sin i}{\sin r}$$

where i is measured = 60° within $\pm 30'$ and r is measured = 50° within $\pm 30'$. What is the precision of μ ?

$$\begin{aligned} \frac{\partial \mu}{\partial i} (\Delta i) &= \frac{\cos i}{\sin^2 r} (\Delta i) = \frac{0.50}{0.77} \left(\frac{0.50}{57} \right) = \pm 0.0057 \\ \frac{\partial \mu}{\partial r} (\Delta r) &= -\frac{\sin i \cos r}{\sin^2 r} (\Delta r) = \frac{0.87 \times 0.64}{(0.77)^2} \times \frac{0.50}{57} = \pm 0.0080 \end{aligned}$$

Then

$$E_\mu = \sqrt{(0.0057)^2 + (0.0080)^2} = \pm 0.0098 = \pm 0.010$$

and

$$\mu = \frac{0.8660}{0.7660} = 1.131 \pm 0.010.$$

Note.— Δi and Δr must be expressed in *radians*.

$$30' = \frac{0.50}{57} \text{ radians.}$$

Note also that an error of 0.8 per cent. in the measured value of i or $(0.5/60 \times 100)$ produces only 0.52 per cent. error $(0.0057/1.1 \times 100)$ in μ , and that a 1.0 per cent. error in r produces a 0.72 per cent. error in μ . The minus sign before the value of $\partial\mu/\partial r(\Delta r)$ means that μ decreases with increasing values of r and vice versa.

The Converse Problem.—Up to this point the problem has been to estimate the precision measure of a final result Q in terms of the precision measures of the component measured quantities. It is as important, if not more so, to be able to determine in advance what deviations are allowable in the components in order to attain a certain desired precision in the final result Q . The consideration of this matter must obviously precede the actual measurement of the component quantities in order that all of them may be measured with the necessary precision, otherwise (1) the whole experiment or test may be rendered worthless by a too inaccurate measurement of one or more of the components, or (2) there may be a considerable waste of time through the use of unnecessarily precise apparatus in the over-precise determination of some component.

Equal Effects.—When there is more than one component the problem may have many different solutions. A *preliminary* solution is often made on the assumption that the errors in the components $q_1, q_2, q_3, \dots, q_n$ produce equal effects on the final result Q . In this case (known as the solution for "equal effects"), if

$$E = \sqrt{e_1^2 + e_2^2 + \dots + e_n^2},$$

for equal effects

$$e_1 = e_2 = e_3 = \dots = e_n$$

and, therefore,

$$E = \sqrt{ne^2}, \text{ or } e = e_1 = e_2 = \dots = e_n = \frac{E}{\sqrt{n}}.$$

Having found the values of $e_1, e_2, e_3, \dots, e_n$, the corresponding values of $\Delta q_1, \Delta q_2, \Delta q_3, \dots, \Delta q_n$ can be computed from

$$e_1 = \frac{\partial Q}{\partial q_1} (\Delta q_1), \quad e_2 = \frac{\partial Q}{\partial q_2} (\Delta q_2), \quad \text{etc.}$$

Typical Example.—The heat conductivity K of a substance is desired within 5 per cent. The quantities to be measured are indicated in the equation

$$K = \frac{Qd}{A\theta T},$$

where Q is the heat transmitted in time T through a piece of the substance having a cross-section A and thickness d when the temperature difference is θ . Suppose the approximate value of K is known to be 6 B.t.u. per hour per square foot per degree Fahrenheit for a thickness of 1 inch. The test is to be run for 8 hours with apparatus whose dimensions are approximately $A = 2$ sq. ft. and $d = \frac{1}{2}$ inch. If $\theta = 70^\circ$ F. approximately, then Q will be nearly 13,000 B.t.u. What one wishes to know is the precision with which the quantities Q , d , A , θ and T must be measured to ensure the desired precision in K . Since 5 per cent. of K equals ± 0.30 , then E_K is given by

$$E_K = \pm 0.30 = \pm \sqrt{\left[\frac{\partial K}{\partial Q}(\Delta Q)\right]^2 + \left[\frac{\partial K}{\partial d}(\Delta d)\right]^2 + \left[\frac{\partial K}{\partial A}(\Delta A)\right]^2 + \left[\frac{\partial K}{\partial \theta}(\Delta \theta)\right]^2 + \left[\frac{\partial K}{\partial T}(\Delta T)\right]^2}.$$

If we assume "equal effects," then

$$\begin{aligned} \frac{\partial K}{\partial Q}(\Delta Q) = \frac{\partial K}{\partial d}(\Delta d) = \frac{\partial K}{\partial A}(\Delta A) = \frac{\partial K}{\partial \theta}(\Delta \theta) \\ = \frac{\partial K}{\partial T}(\Delta T) = \pm \frac{0.30}{\sqrt{5}} = \pm 0.13 \end{aligned}$$

from which the values ΔQ , Δd , ΔA , $\Delta \theta$ and ΔT may be computed. For example,

$$\begin{aligned} \frac{\partial K}{\partial Q}(\Delta Q) = \frac{d}{A\theta T}(\Delta Q) = \frac{0.50}{2 \times 70 \times 8}(\Delta Q) = \frac{0.30}{\sqrt{5}} = \pm 0.13 \\ \therefore \Delta Q = \frac{2 \times 70 \times 8}{0.50} \times 0.13 = \pm 290 \text{ B.t.u.} \end{aligned}$$

Similarly, $\Delta d = \pm 0.011$ inch, $\Delta A = \pm 0.045$ sq. ft.,

$\Delta \theta = \pm 1.5^\circ$ F. and $\Delta T = \pm 0.18$ hour = ± 11 minutes.

An inspection of these results shows that there will be no difficulty in measuring the area, thickness and time with a

precision so much higher than the solution by "equal effects" demands, that the errors which they will introduce in the final value of K will be negligible. Consequently a greater latitude in the measurement of Q and θ is permissible. A new solution of the problem could now be made on the basis that Q and θ shall produce "equal effects" on the precision of the final result, the effect of the other errors being considered negligible. For such a solution

$$\frac{\partial K}{\partial Q} (\Delta Q) = \frac{\partial K}{\partial \theta} (\Delta \theta) = \frac{0.30}{\sqrt{2}} = \pm 0.21$$

from which $\Delta Q = \pm 470$ B.t.u. approximately, and $\Delta \theta = \pm 2.5^\circ$ F. approximately.

Unequal Effects.—A study of the apparatus and the method of measuring the two factors Q and θ may, however, show that it is actually possible to measure one factor, θ , with such precision that it will produce less effect on the final result than the error in the other factor. Let us, for example, assume that the error in the measurement of Q will produce twice as great an effect in K as that produced by the error in the temperature measurements. Under such circumstances, it would be wiser to estimate their required precision measures in the following manner.

Assume that

$$\frac{\partial K}{\partial Q} (\Delta Q) = 2 \frac{\partial K}{\partial \theta} (\Delta \theta),$$

then the equation

$$E_K = \pm 0.30 = \sqrt{\left[\frac{\partial K}{\partial \theta} (\Delta \theta) \right]^2 + \left[\frac{\partial K}{\partial Q} (\Delta Q) \right]^2}$$

becomes

$$E_K = \pm 0.30 = \sqrt{5 \left[\frac{\partial K}{\partial \theta} (\Delta \theta) \right]^2}$$

and, therefore,

$$\frac{\partial K}{\partial \theta} (\Delta \theta) = \frac{\pm 0.30}{\sqrt{5}} = \pm 0.13$$

whence

$$\Delta \theta = \pm 1.5^\circ \text{ F.}$$

and, similarly,

$$\Delta Q = \pm 580 \text{ B.t.u.}$$

Though the preceding treatment is a possible and plausible one, in actual practice where the factors are as few as two, as

was assumed above, some risk is taken in allowing for the probability of the two errors neutralizing each other as is inherent in the relation

$$E = \sqrt{e_1^2 + e_2^2}.$$

It may, therefore, be advisable, in order to obtain an estimate of the largest allowable precision measure of the two components that will give the desired precision in the final result, to assume that the two errors act in the same direction, that is to write

$$\frac{\partial K}{\partial Q}(\Delta Q) + \frac{\partial K}{\partial \theta}(\Delta \theta) = \pm 0.30.$$

Since

$$\frac{\partial K}{\partial Q}(\Delta Q) = 2 \frac{\partial K}{\partial \theta}(\Delta \theta),$$

it follows that

$$\frac{3}{2} \frac{\partial K}{\partial Q}(\Delta Q) = \pm 0.30$$

The solution gives $\Delta Q = \pm 450$ B.t.u. and $\Delta \theta = \pm 1.2^\circ$ F.

If θ is a temperature difference ($t_2 - t_1$), assuming equal effects and the theory of probabilities, the average thermometer readings over the 8 hours must be known to

$$\frac{\pm 1.2}{\sqrt{2}} = \pm 0.85^\circ \text{ F.}$$

To be absolutely safe one might neglect the possibility of the error in t_2 neutralizing that in t_1 and assume them to be additive. This would require the precision of t_2 and of t_1 to equal

$$\frac{\pm 1.2}{2} = \pm 0.6^\circ \text{ F.}$$

Suppose furthermore that the heat is produced by a heating coil taking approximately 30 amperes at 16 volts. The heat generated is given by the relation

$$IET = 30 \times 16 \times 28,800 \text{ joules} = 13,000 \text{ B.t.u. (approx.)}$$

This quantity of heat is desired to ± 450 B.t.u., *i. e.*, to 3.5 per cent. Assuming no appreciable error in measuring the time T (8 hours), the precision with which I and E must be measured will depend upon whether the precision measure p.m. of Q is

taken equal to $\sqrt{e_1^2 + e_2^2}$, where

$$e_1 = e_2 = \frac{\text{p.m.}}{\sqrt{2}},$$

or whether it is considered advisable to determine the maximum allowable errors by assuming the errors in I and E to have the same sign, *i. e.*, p.m. of $Q = e_1 + e_2$. In the latter case I must be measured within $3.5/2 = 1.7$ per cent. or within ± 0.51 ampere, and E must be measured within 1.7 per cent. or ± 0.27 volt. That is the ammeter and voltmeter must read accurately within these limits and must also be sensitive within these limits.

Weighting Measurements of Unequal Precision.—When observations on a quantity are made under similar conditions and are thought to be equally probable, it has been shown that the most probable value of the measured quantity is the arithmetical mean.

It frequently happens that it is necessary to determine the best representative value of a quantity from a series of results which have been obtained under *unlike conditions* (*e. g.*, by different observers of unequal skill, by different methods, by means of apparatus of unequal sensitiveness, etc.), and which have not the same precision. Some of these results deserve more confidence or greater *weight* than others. In this case it is first necessary to assign *relative weights* to the observations, so that, in taking the average, the more precise measurements may be given a proportionally greater *weight* than the less precise measurements. The weight to be assigned to a result is a number to indicate the trustworthiness of that result as compared with others with which it is to be combined.

In the absence of other information regarding the details of the experimental work, the precision measure *may* serve as an indication of the weights to be assigned in the following manner. Suppose the results of different sets of observations are $Q_1, Q_2, Q_3, \dots, Q_n$ and the respective precision measure of these independent results are $p_1, p_2, p_3, \dots, p_n$. Let $w_1, w_2, w_3, \dots, w_n$ be the weights assigned to the separate results. If, for example, Q_1 is given the weight 5 and Q_3 the weight 10, it would mean that in determining the most probable value of the quantity Q , Q_1 is to be counted 5 times while Q_3 is to be counted 10 times. It can be shown (see Palmer, Theory of Measurements, Chap.

VI.) that the weights assigned should be inversely proportional to the squares of the precision measures, any arbitrary weight being assigned to one of the given measurements. That is

$$w_2 = w_1 \frac{p_1^2}{p_2^2}; \quad w_3 = w_1 \frac{p_1^2}{p_3^2}; \quad \dots; \quad w_n = w_1 \frac{p_1^2}{p_n^2}.$$

This method of computing weights assumes the quantities free from constant errors and mistakes. If this condition is met, then

$$Q = \frac{w_1 Q_1 + w_2 Q_2 + \dots + w_n Q_n}{w_1 + w_2 + \dots + w_n} \quad (1)$$

is the most probable value of the quantity. For example, various experimenters have obtained the following results on the velocity of light (Wright and Hayford, p. 57).

$$\begin{aligned} Q_1 &= 298,000 \pm 1000 \text{ km. per sec.} \\ Q_2 &= 298,500 \pm 1000 \quad " \quad " \quad " \\ Q_3 &= 299,990 \pm 200 \quad " \quad " \quad " \\ Q_4 &= 300,100 \pm 1000 \quad " \quad " \quad " \\ Q_5 &= 299,930 \pm 100 \quad " \quad " \quad " \end{aligned}$$

Assuming the weight of the first value to be unity, the second would be one, the third

$$\left[1 \times \left(\frac{1000}{200} \right)^2 \right] = 25,$$

the fourth one, and the fifth

$$\left[1 \times \left(\frac{1000}{100} \right)^2 \right] = 100.$$

The most probable value of Q would then be by equations (1)

$$\begin{aligned} Q &= \frac{1 \times Q_1 + 1 \times Q_2 + 25 \times Q_3 + 1 \times Q_4 + 100 \times Q_5}{1 + 1 + 25 + 1 + 100} \\ &= 299,916 \text{ km. per sec.} \end{aligned}$$

The weight of this mean value is

$$w = w_1 + w_2 + \dots + w_n = 128,$$

and the precision measure of the quantity is

$$p = p_k \frac{\sqrt{w_k}}{\sqrt{w}}.$$

Using the third quantity in the list, the value for p becomes

$$p = 200 \sqrt{\frac{25}{128}} = \pm 88 \text{ km. per sec.}$$

Another illustration of the need of weighting results can be seen in the problem of finding the mean efficiency of a power plant during the period of one year. A value may be found by finding the mean efficiency for each month of the year and then taking the average of these twelve values as the mean efficiency. A *more representative value* of the *mean efficiency* would be found by basing the computation on the total output and the total input for the year, since the business done in different months may differ widely and the values deserve to be weighted according to the business done.

As an illustration of the weighting of observations see note at the end of Exercise 6.

General Remarks

Limitations of the Precision Theory.—It must be borne in mind that the precision measure is not a guarantee of accuracy, but a means of estimating the probability of a result. It often happens that there are so many unknown constant errors or mistakes in theory underlying an experimental result that one almost hesitates to test for probability by the regular theoretical methods. However, these methods are the best at our disposal. The precision measure is a better estimate of reproducibility than of accuracy. It must also be remembered that the whole theory of probabilities is based on the assumption of an infinite number of trials. When the number of observations is small, the A.D. of the A.D. itself becomes large. The best check on the accuracy of a result is to compare it with results obtained by other observers using entirely different methods. The discrepancy between the precision measure and the accuracy has led many experimenters even at our Bureau of Standards to scorn such terms as "probable error," "A.D.," and "square root of the sum of the squares" in their work. In this course the square root of the sum of the squares of the $\pm$ errors will be used rather than the sum of the errors not because we are sure the actual error will not be so great as the sum of the errors, but because we think it *not probable*.

Guarantees.—When an instrument maker sends a guarantee with an instrument he usually takes into account the various possible errors, adds them up and then uses some factor to determine the limits within which he is willing to make the guarantee, the factor being a safety factor determined by his past experience. For example, in catalogues are found statements like this: "Adjusted to 1/10 per cent. and guaranteed to 1/5 per cent."

Tolerances.—This term is used to denote the outside errors which will be tolerated. The size of this error is usually such that with good apparatus it can be detected without any doubt. For example, a spring balance may be required to register 2 lbs. with a "tolerance" of 1/16 oz., or a 10 kg. weight may have a "tolerance" of 2,000 mg., or a 6 ft. length may have a "tolerance" of 3/16 inch., or a bushel may have a "tolerance" of 25 cubic inches. There is no definite numerical relation between the tolerance and the precision measure of the calibrated standard. The student should form the habit of analyzing the experimental method and apparatus with reference to precision possibilities. This should be done in a rough way. He should set for each of the quantities measured an outside limit of error or tolerance which he should not exceed. He should then compute the effect of such errors on the final result, thus obtaining an estimate of the maximum allowable error.

Absence of Accidental Errors.—It sometimes happens that no accidental variations appear in a set of observations. This may be the result of inaccurate observations, a lack of sensitivity in the instruments used, or a failure to make the observations independent of one another. For example, suppose the distance between two lines is to be measured. A length scale is set on the two lines and the fractional divisions on the scale estimated at both points. If the scale is not reset for subsequent readings, no accidental deviations will appear. This does not mean that the observations are perfect, rather that the measurements were not independent of one another.

Curve Plotting.—A curve plot is usually the most condensed form of presenting experimental data and also the most illuminating. It should, therefore, always be plotted with care. The following suggestions will serve as a guide to good curve plotting and Fig. B will illustrate the points mentioned.

1. Plot the independent variable as abscissa and the dependent variable as ordinate. For example, the time of year as abscissa and the price of wheat as ordinate.

2. Choose the scale of the two variables so as to make the general slope of the curve about 45° .

3. Plot both variables to such scale that all significant figures in the data can be read from the graph.

4. Choose scale units that can be easily read, preferably even numbers.

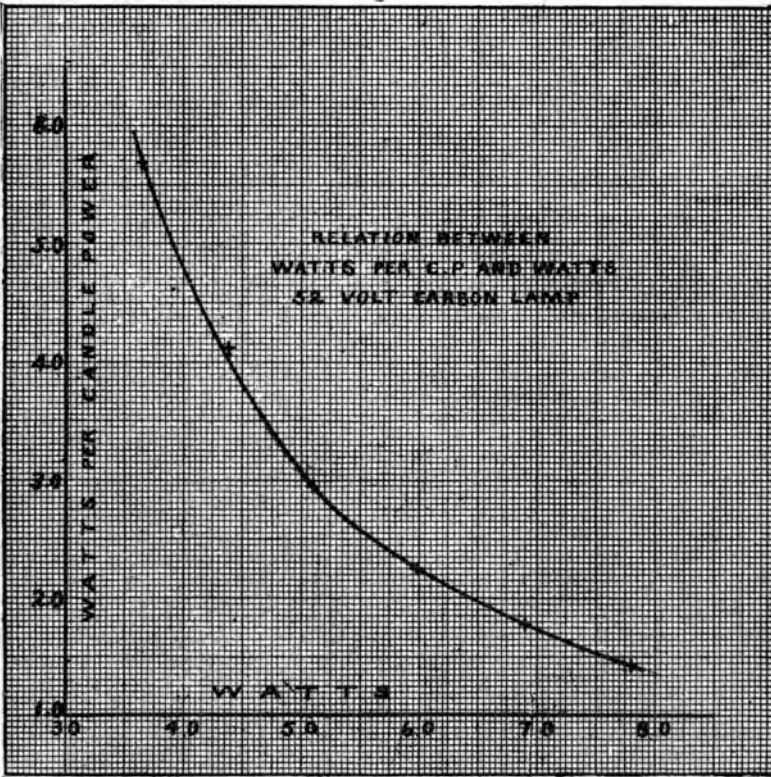


FIG. B.

5. It is not always necessary to have the origin on the plot.

6. Indicate the points on the plot clearly with crosses or circles.

7. If the relation between the variables is a continuous one, draw through the set of points a representative smooth curve with a French or flexible curve. If the relation is evidently a

straight line relation, draw a representative line with a straight edge, leaving the points as evenly distributed as possible on both sides of the line.

8. Label the axes and state the units chosen.

9. Give a title to the curve as a whole.

The data plotted in Fig. B are the following,

Watts.	Watts per C.P.
36.5	5.68
43.9	4.11
51.0	2.95
59.8	2.25
69.0	1.78
78.0	1.44

For an excellent discussion on graphical methods, see Goodwin's Precision of Measurements and Graphical Methods.

Solution of Illustrative Problems

Problem 1.—In timing a train it is observed that it passes over a measured distance between 27 minutes 15 seconds past the hour, and 28 minutes 5 seconds past the hour. The time is, then, given by (28 min. 5 sec. — 27 min. 15 sec.) = 50 sec. (1) If these times can each be read from the watch within 1 second, how dependable is their difference?

Here $M = m_1 - m_2$, $(\Delta m_1) = 1$ sec. and $(\Delta m_2) = 1$ sec. The resultant effect of both is

$$E = \sqrt{(1.0)^2 + (1.0)^2} = \sqrt{2.0} = \pm 1.4 \text{ sec.},$$

or 2.8 per cent. uncertainty in the 50 seconds.

(2) With the same watch, how precisely could a time interval of 7 minutes be determined? Ans. 1.4 sec. or 0.33 per cent.

Converse Problem.—(1) To determine the above time interval of 50 secs. within 1 second the readings at the beginning and end would each need to be accurate within ± 0.71 second, assuming "Equal Effects," for

$$e_1 = e_2 = \frac{1.0}{\sqrt{2}} = \pm 0.71 \text{ second.}$$

But $e_1 = (\Delta m_1)$ and $e_2 = (\Delta m_2)$, therefore

$$(\Delta m_1) = (\Delta m_2) = \pm 0.71 \text{ sec.}$$

Problem 2.—The modulus of elasticity of a cylindrical wire of length L , cross-section A , which when loaded with a weight W is elongated by an amount x , is given by the expression

$$M = \frac{LW}{Ax}$$

If,

$$\begin{aligned} L &= 200.11 \text{ cm., A.D.} = \pm 0.34 \text{ cm.} \\ W &= 10 \text{ kg., accurate to 10 grams} \\ A &= 0.007853 \text{ cm}^2, \text{ correct to } \pm 0.30 \text{ per cent.} \\ x &= 0.12546 \text{ cm., measured within } \pm 0.00025 \text{ cm.} \end{aligned}$$

how precisely is the determination of M ? Also compute the value of M using the proper number of significant figures.

Solution.—The percentage deviations in the *components* are:

$$\begin{aligned} \frac{100(\Delta L)}{L} &= 0.17 \text{ per cent.}; & \frac{100(\Delta W)}{W} &= 0.10 \text{ per cent.}; \\ \frac{100(\Delta A)}{A} &= 0.30 \text{ per cent.}; & \frac{100(\Delta x)}{x} &= 0.20 \text{ per cent.} \end{aligned}$$

Consequently the percentage deviations in the value of M are:

$$\begin{aligned} \frac{100e_L}{M} &= 0.17 \text{ per cent.}; & \frac{100e_W}{M} &= 0.10 \text{ per cent.}; \\ \frac{100e_A}{M} &= 0.30 \text{ per cent.}; & \frac{100e_x}{M} &= 0.20 \text{ per cent.} \end{aligned}$$

Hence

$$\frac{100E}{M} = \sqrt{(.17)^2 + (.10)^2 + (.30)^2 + (.20)^2} = \pm 0.41 \text{ per cent.},$$

i. e., M is known with a precision of 0.41 per cent.

To which component should attention first be given in attempting to improve this precision?

The computed value of M is:

$$M = \frac{200.11 \times 10.000}{0.007853 \times 0.12546} = \frac{2001.1}{9.8524 \times 10^{-4}} = 2.031 \times 10^6 \frac{\text{kg}}{\text{cm}^2} \pm 0.008 \times 10^6 \frac{\text{kg}}{\text{cm}^2}$$

Problem 3.—The volume of a right circular cylinder is found by measuring its diameter d and length l . $V = \frac{1}{4}\pi d^2 l$. Compute the volume and its precision measure given the following data:

$$\begin{aligned} d &= 7.203 \text{ cm.} & \text{A.D.} &= \pm 0.024 \text{ cm.} \\ l &= 4.408 \text{ cm.} & \text{A.D.} &= \pm 0.022 \text{ cm.} \end{aligned}$$

Solution: The percentage uncertainty in d is

$$\frac{100(\Delta d)}{d} = \frac{100 \times 0.024}{7.2} = 0.33 \text{ per cent.}$$

Since V is proportional to d^2 , this error produces in V an uncertainty of $2 \times 0.33 = 0.66$ per cent.

The percentage uncertainty in l is

$$\frac{100(\Delta l)}{l} = \frac{100 \times 0.022}{4.4} = 0.50 \text{ per cent.}$$

and this produces a like uncertainty of 0.50 per cent. in V . The uncertainty in V , due to both components, is then

$$\frac{100E}{V} = \sqrt{(0.66)^2 + (0.50)^2} = \pm 0.83 \text{ per cent.}$$

$$V = \frac{1}{4} (3.1416) \times (7.203)^2 \times 4.408 = 179.6 \text{ cm}^3 \pm 1.5 \text{ cm}^3.$$

Converse Problem.—Suppose the volume V is desired within ± 0.50 per cent., what precision is required in the factors assuming “equal effects”?
Solution.—

$$\frac{100E}{V} = \sqrt{\left(\frac{100e_d}{V}\right)^2 + \left(\frac{100e_l}{V}\right)^2} = \pm 0.50 \text{ per cent.}$$

For equal effects

$$\frac{100e_d}{V} = \frac{100e_l}{V} = \frac{0.50}{\sqrt{2}} = 0.35 \text{ per cent.}$$

But

$$\frac{100e_d}{V} = 2 \times \frac{100(\Delta d)}{d} = 0.35 \text{ per cent.}$$

Hence d must be measured within $\frac{1}{2}$ of 0.35 or ± 0.18 per cent., or within ± 0.013 cm. Again

$$\frac{100e_l}{V} = \frac{100(\Delta l)}{l} = 0.35 \text{ per cent.}$$

Hence l must be measured within ± 0.35 per cent. or within ± 0.015 cm.

Problem 4.—The specific gravity of a solid is determined with a specific gravity flask and the result is desired within ± 0.20 per cent. The equation to be used is

$$S = \frac{m}{m + m_2 - m_3}.$$

The following results are obtained:

$$m = 14.3844g; m_2 = 75.6657g; m_3 = 84.3590g.$$

How precisely must these weighings be made? Compute S and express with the proper number of figures.

Solution: A rough calculation on the slide rule gives $S = 2.5$. Now

$$\pm 0.20 \text{ per cent of } S = \pm 0.0050,$$

which is the permissible error in S . Assuming equal effects from the three factors gives an allowable effect from each of them equal to

$$\frac{0.0050}{\sqrt{3}} = \pm 0.0029$$

$$\frac{\partial S}{\partial m} (\Delta m) = \frac{m_2 - m_3}{(m + m_2 - m_3)^2} (\Delta m) = \frac{-8.7}{(5.7)^2} (\Delta m) = \pm 0.0029$$

whence $(\Delta m) = \pm 0.011g$.

$$\frac{\partial S}{\partial m_2} (\Delta m_2) = \frac{-m}{(m + m_2 - m_3)^2} (\Delta m_2) = \frac{-14}{(5.7)^2} (\Delta m) = \pm 0.0029$$

whence $(\Delta m_2) = \pm 0.0066$ g.

$$\frac{\partial S}{\partial m_2} (\Delta m_2) = \frac{+m}{(m + m_2 - m_3)^2} (\Delta m_2) = \pm 0.0029$$

whence $(\Delta m_3) = \pm 0.0066$ g.

$$S = \frac{14.3844}{14.3844 + 75.6657 - 84.3590} = 2.5268 \pm 0.0050$$

Why is it that the precision required for m is not as great as for m_2 and m_3 ? What is the significance of the minus sign for the value of $\partial S/\partial m_2$ and the + sign for $\partial S/\partial m_3$?

PRECISION PROBLEMS

- (a) When is a figure experimentally significant?
(b) How many significant figures have the following numbers, assuming them to be correctly expressed? 38100; 4730.0; 5203.6; 013.072; 2.050; 0.00830; 0.0091.
- Express the following quantities with the proper number of significant figures: 3.456 ± 0.18 ; 746000.4 ± 20 ; 0.002654 ± 0.000087 ; 6523.997 ± 0.286 ; 12.67 good to 5 parts in 1000; 7.4683 reliable to 0.20 per cent. How many significant figures has each number?
- (a) Add the numbers in problem 2.
(b) Subtract the last number from the fourth in problem 2.
(c) State the rule for significant figures in addition and subtraction.
- In computing the moment of inertia of a flywheel with a slide rule the following results were obtained: the moment of inertia of the rim 9270 units, of the spokes 243.0 units, of the hub 13.67 units. What is the total moment of inertia? How many significant figures should be used in expressing the result?
- Express as a power of 10 the following numbers to show the significant figures:
Velocity of light = 299017000 ± 88000 m per sec.
Mechanical equivalent of heat = 41830000 ± 40000 ergs per calorie.
Electrochemical equivalent of H = 96540 ± 10 coulombs per gram.
Electronic charge = 477500×10^{-18} c.g.s.e.s. units good to 0.50 per cent.
- If the following numbers are uncertain in the last place of figures, make the following computations using the correct number of figures:

- Add 372.8254, 25.61, 0.43789.
- Product of $24.00 \times 11.3 \times 3.1416$.
- Quotient $\frac{3887.2 \times 2.146}{25.4}$.

- The distance between two points was measured in three sections, the measurements being made in feet and tenths of a foot. The values obtained were: 23.4, 37.8 and 18.8 feet. What is one third the distance between the points.
- It is found by experiment that one ampere deposits copper at the rate of .000329 gram per second. How many coulombs will be required to deposit 31.53 grams?

9. The value of the electronic charge is $e = 1.6 \times 10^{-20}$ electromagnetic units. The mass of an electron is 8.9×10^{-28} gram. Compute the ratio e/m .

10. (a) What is the precision of a multiplication made on an ordinary slide rule? (b) Multiply 4.30 by 22.9 on a slide rule. Write result so as to indicate the precision of your result.

11. Use a slide rule to compute the area of a rectangle 5375 inches by 2445 inches. Add to this the following areas; one third of the area of a rectangle 4000 inches square, one sixth of the area of a rectangle 500 by 1600 inches, and one eighth of the area of a rectangle 200 by 80 inches.

12. The equation for a Weston normal cadmium cell is $E = 1.01830 - 0.0000406 (t - 20) - 0.00000095 (t - 20)^2 + 0.000000001 (t - 20)^3$.

When would you use the first temperature correction term? When the second? When the third?

13. A steel vernier caliper, calibrated at 60°F ., reads to thousandths of an inch. Would you make a temperature correction on a two inch length measured with the caliper at 0°F . if the temperature coefficient of expansion for steel is 6.6×10^{-6} referred to the F . scale?

14. A series of nine observations on the angle of a prism gives a mean value of $67^\circ 13' 20''$. The individual observations differ from the mean by amounts whose average is $45''$. Find a.d., A.D. and the percentage deviation of the mean.

Correction for a constant error in the scale changes this result to $67^\circ 14' 40''$. This correction is reliable within $10''$. Find the percentage precision of the final result.

$$15. M = 5m_1 + 3m_2 + m_3,$$

If $m_1 = 15.039 \text{ gm.}$, measured to 0.30 per cent.

$$m_2 = 3.0264 \text{ gm.}, \text{ A.D. } 0.0060 \text{ gm.}$$

$$m_3 = 20.14 \text{ gm.}, \text{ measured to 1 part in 200}$$

how reliable is M in grams? In per cent.?

What would be the reliability of $5m_1 - 3m_2 - m_3$ in grams? In per cent.?

16. In an atomic weight table the atomic weight of sulphur is given 32.07, that of oxygen 16 (exact) and of hydrogen 1.008. If the electrochemical equivalent of hydrogen is 96540 ± 10 coulombs per gram, compute the electrochemical equivalent of SO_4 using the proper number of figures.

$$17. M = \frac{a \times b}{c} \text{ grams.}$$

$$a = 120.64 \text{ with its A.D. } = 0.12.$$

$$b = 0.007865 \text{ known within 1.4 per cent.}$$

$$c = 10.0543 \text{ with its A.D. } = 0.0025.$$

Compute M using the proper number of significant figures throughout the multiplication and division. What is the actual and what the percentage precision of M ? Express M with the proper number of significant figures. Also solve for M using logarithms.

18. The velocity of an automobile is determined over a 1000 ft. trap. If the stop watches used are good to 0.2 second at the start and 0.2 second at the stop (i. e. time = $t_2 - t_1$, where $\delta_2 = \delta_1 = 0.2$ second), how closely can the time (about 25 seconds) be determined? What percentage error would this cause in the speed as computed? What uncertainty (in miles per hour) would this represent?

19. The power factor in an alternating current circuit is given by

$$\text{P.F.} = \frac{\text{actual watts}}{\text{apparent watts}} = \frac{W}{EI}$$

The power W is 700.0 watts, read by a wattmeter which is dependable at this part of the scale within 4.0 watts, the e.m.f. $E = 115.0$ volts by a voltmeter which can be read within 0.5 volt, and the current $I = 10.05$ amperes, liable to an error of 0.10 ampere in its reading. Compute the P.F., using the proper number of significant figures throughout. Find its precision in per cent.

20. An electromotive force is determined by comparison by the potentiometer method, the result being $e_2 = \frac{r_2}{r_1} e_1$. If the resistances r_1 and r_2 are each known within 0.04 per cent. and the e.m.f. of the standard cell, e_1 , within 0.03 per cent., how dependable is the resulting value of e_2 ? How many significant figures should be used in computing it?

21. The e.m.f. of a Clark cell is $E_t = 1.4328 [1 - 0.00083 (t - 15^\circ)]$ where t is the centigrade temperature. If t is about 25°C. and can be read within 0.2° , how accurately can the e.m.f. of the cell be computed? What is the percentage error?

22. The area of a circle whose diameter is about 6 inches is desired, (1) within a deviation of 0.045 sq. in., (2) within 0.10 per cent. How closely must the diameter be measured in each case, and how many significant figures should be used in the value of π in the computation?

23. The height of a barometer, corrected for temperature, is given by

$$H_0 = H_t \cdot [1 - (\beta - \alpha)t]$$

If $(\beta - \alpha) = 0.000163$, and $H_t =$ about 760 mm., what error is introduced into H_0 by an error of 0.3°C. in t when t is about 30°C. What if t is about 15°C.

If this uncertainty in t is to cause no error in H_0 greater than 0.100 mm. (when H_t is about 760 mm.), how closely must the thermometer be read?

24. The output (in horse-power) of a motor is determined by means of a friction brake. Horse-power (h.p.) = $2\pi LNP/33000$, where L = radius of pulley, N = number of revolutions per minute, P = load in pounds.

$L = 2.017$ ft. measured within 0.20 per cent.

$N = 601.3$ r.p.m. within 2 r.p.m.

$P = 20.08$ lbs. measured within 0.14 lb.

What is the percentage precision in the value of the h.p.? How many significant figures should be used in computing it?

25. The horse-power of a gas engine is given by the formula

$$\text{H.P.} = LEAP/33000$$

where L is the length of stroke, E is the number of effective or working strokes per minute, A is the area of the piston ($A = 1/4\pi d^2$) and P is the mean effective pressure.

If $L = .9001$ ft., A.D. = 0.0045 ft.

$E = 100$ strokes per minute, measured within 1 stroke

$d = 6$ inches, within 0.25 per cent., and

$P = 60$ lbs. per sq. in., known within 2.0 lbs. per sq. in.,

find the percentage deviation in H.P. How many significant figures should be used in the computation? Which, if any, of the above measurements have been made too precisely? Which should be improved?

26. The rate of flow (F) of water from an orifice in the side of a tank is given (in cubic feet per second) by $F = KA\sqrt{2gH}$, where K is a constant depending on the shape of the opening, A is its area, and H is the head of water. These quantities have the values given below, and have been measured with the precisions stated. Compute the flow, using the proper number of significant figures. Find the reliability of this result in cubic feet per second.

$H = 4.324$ ft., known within 0.016 ft.

$g = 32.164$ ft. per sec.², known within 1 part in 500.

$A = 0.03142$ sq. ft., known within 1/5 of 1 per cent.

$K = 0.6065$, accurate to 1/3 of one per cent.

27. It is desired to determine the value of the constant K for a certain orifice by collecting and weighing the water that passes through it in two minutes. According to the above formula we have

$$K = \frac{W}{DA\sqrt{2gH}}$$

where W = weight of water (in lbs.)

D = density of water (in lbs. per cu. ft.)

t = time of flow (in seconds).

If A , g , H , have the same values and precisions as given in problem (26)

$t = 119.2$ seconds, measured within 0.2 second

$W = 2245$ lbs., within about 9 lbs., and

$D = 62.30$ lbs. per cu. ft., known within 0.01 lb. per cu. ft., how dependable is the resulting value of K ? How many significant figures should be used in its computation?

28. The coefficient of rigidity of a wire is determined by means of a torsion pendulum, using a disk whose moment of inertia is computed from the formula

$$I_d = \frac{MR^2}{2}.$$

The coefficient of rigidity is computed from the formula

$$n = \frac{8\pi I_d l}{t_d^2 r^4}.$$

The measured quantities have the following values:

M = mass of disk = 4001.0 grams known within 4.0 grams.

R = radius of disk = 26.031 cms. measured within 0.052 cm.

t_d = 2.712 seconds known within 1 per cent.

l = length of wire = 179.30 cms. known within 1.8 millimeters.

r = radius of the wire. Nine measurements of its *diameter* give an average value of 0.2430 cm.; the individual readings differ from this mean by amounts whose average is 0.0012 cm.

(1) How precise will be the value of I_d computed from the above data? (2) Find the precision measure of r . (3) What percentage deviation in n will be caused by using this value of I_d ? By the uncertainty in r ? (4) Compute n , and state the rule for the number of significant figures used. (5) What is the precision measure of n ?

29. The resistance R_0 of a certain coil at 0° is 30.010 ohms, known within 0.015 ohm. Its resistance at any other temperature t is given by the relation

$$R_t = R_0(1 + \alpha t)$$

where α , the temperature coefficient, is 0.00491, known within 1.0 per cent. If a thermometer is used which can be depended upon within 0.3° C., how precisely (expressed (1) in percentage, (2) in ohms) can the resistance at 15° be computed? At 45° ?

30. The input of a motor is measured by an ammeter and a voltmeter. Input (in h.p.) = $IE/746$.

If the current I is 45.05 amperes, known within 0.15 ampere, and the e.m.f. E is 116.3 volts, known within 0.35 volt, how precisely can the input be found? If the output were known within 0.80 per cent., how closely could the efficiency (= output/input) be found?

31. The e.m.f. induced at a given instant in a conductor cutting lines of magnetic force is given by $e = \frac{H L v \sin \theta}{10^8}$ volts, where H = magnetic field strength, L = length of conductor, v = linear speed, θ = angle between the velocity of the moving conductor and the direction of the magnetic flux.

If $H = 5000 \frac{\text{lines}}{\text{sq. cm.}}$ known to 2.0 per cent., $L = 25.000$ cms. with A.D. = 0.035

cm., $v = 500.0 \frac{\text{cm.}}{\text{sec.}}$ known to 0.50 per cent., and $\theta = 30^\circ$ approximately known to 2.0 per cent., compute the percentage precision in e , also the uncertainty in e in volts.

32. It is desired to compute the period T of a pendulum from its length and the value of g

$$T = 2\pi \sqrt{\frac{l}{g}}$$

If $l = 107.31$ cm., A.D. = 0.20 cm. and

$g = 980.63$ cm./sec.² known within 1/10 per cent., what will be the uncertainty in T (1) in per cent., (2) in seconds? How many significant figures should be used in computing T ?

33. The velocity of flow of water through a pipe is determined by inserting a Venturi meter:

$$V_2 = K \sqrt{\frac{2g(H_1 - H_2)}{1 - \left(\frac{A_2}{A_1}\right)^2}} \text{ ft. per second}$$

where V_2 is the velocity at the point 2, A_2 and A_1 are the cross-sections of the

tube at points 2 and 1 respectively, H_2 and H_1 the pressures at these points measured in feet of water by manometers properly inserted, and K is a constant equal to about 0.98. Suppose H_2 and H_1 can each be measured within 0.05 ft., and $H_1 - H_2$ is about 2 ft.; A_2/A_1 is 0.5000, accurate within one per cent.; g is known within 1/5 of one per cent.; and K within 1/3 of one per cent. What is the effect on V_2 of the errors in the quantities ($H_1 - H_2$), A_2/A_1 , g , K , respectively? What is the percentage precision of V_2 resulting from all of these errors?

34. In determining the value of " g " by the method of coincidences the formula for the half-period of the pendulum is

$$T = \frac{N}{N - n} \times (\text{rate of the clock}).$$

N is found to be 1500 seconds ± 10 seconds, and n , the number of coincidences, is 15 (no error).

What is the precision of T assuming the standard clock to be correct? Suppose the standard clock loses a half-minute each day, what is the true value of the half-period of the pendulum used above? What error in percent. would a failure to make correction for this error in the standard clock introduce in the computed value of " g "?

35. The following equation (see Exercise 24) holds for a change of state:

$$\frac{dp}{dt} = \frac{L}{T(v_2 - v_1)}.$$

$$\left(\frac{dp}{dt}\right) = 35,000 \frac{\text{dynes}}{\text{cm}^2} \text{ per } ^\circ\text{C with an A.D.} = 2.0 \text{ per cent.}$$

$$L = 22.638 \times 10^9 \text{ ergs per gram with an A.D.} = 0.5 \text{ per cent.}$$

$$T = 373.00^\circ \text{ C. absolute} \quad \text{" " " " } = 0.2 \text{ per cent.}$$

$$V_1 = 1.000 \text{ cm.}^3 \quad \text{" " " " } = 0.1 \text{ per cent.}$$

Assuming the above quantities to be independent of one another, solve for v_2 using the proper number of significant figures in the computations and give the precision measure of v_2 in per cent.

36. In using the standard resistance form of potentiometer to measure the e.m.f. e_2 of an unknown cell, the following data were obtained:

$$e_2 = \frac{R + 5000}{R + 183} e_1.$$

R is a 10,000 ohm coil good to 0.050 per cent. = 5.0 ohms.

The 5,000 ohm coil is good to 0.050 per cent. = 2.5 ohms.

The 183 ohm coil is good to 0.050 per cent. = 0.09 ohm.

The standard cell $e_1 = 1.01830$ volt ± 0.00010 volt.

What is the precision of e_2 in volts and in per cent.? Compute value of e_2 using the proper number of significant figures.

37. Suppose the volume of a vessel V is given by the relation

$$\begin{aligned} V &= (abc + def + ghi) \text{ c.c.} \\ &= (1 \times 2 \times 3 + 4 \times 5 \times 6 + 7 \times 8 \times 9) \text{ c.c.} \\ &= (6 + 120 + 504) = 630 \text{ c.c.} \end{aligned}$$

If V is desired within 3 per cent., find the precision for each of the nine quantities assuming that they produce equal effects on V .

38. The index of refraction of a solid by the prism method is given by

$$\mu = \frac{\sin \frac{\Delta + \alpha}{2}}{\sin \frac{\alpha}{2}}.$$

Suppose that $\Delta = 50^\circ \pm 10'$ and $\alpha = 60^\circ \pm 10'$, what is the precision of μ ? Compute value of μ using the proper number of significant figures.

39. The area A of a triangle whose base (b) is about 10 cm. and height (h) about 32 cm. is to be determined within 0.30 per cent.

$$A = \frac{1}{2}bh$$

Assuming equal effects, what are the allowable percentage deviations in b and h ? What are they in centimeters?

If $b = m_1 - m_2$ where m_1 is about 21 and m_2 is about 11, what are the deviations (in cm.) allowable in m_1 and m_2 to give the above precision in b ? What if m_1 were 71 and m_2 were 61?

40. The current (I) in a circuit is measured from the voltage (E) across a known resistance (R) in the circuit.

$$I = E/R$$

$E = 0.3132$ volt, liable to an error of 0.0016 volt and $R = 1.0061$ ohm measured within 0.0040 ohm.

Find I , using proper number of significant figures.

Find the precision of I , in per cent., in amperes. If it were desired to determine I within 0.5 per cent., how precisely would E have to be measured if R remained as above?

41. The volume of a square pyramid is $V = \frac{1}{3}l^2h$, where l = length of side of base and h = height.

$$l = 3.0164 \text{ in. measured within } 0.0050 \text{ in.}$$

$$h = 6.019 \text{ in. measured within } 0.20 \text{ per cent.}$$

What is the percentage uncertainty in V due to the possible error in l ? Due to that in h ? Due to both? How many significant figures should be used in the computation of V ? If this volume is to be found within 0.28 per cent., what deviations (in inches) are allowable in h and l according to "equal effects"?

42. It is desired to determine g to 0.10 cm. per sec.² by means of a pendulum.

$$g = \frac{\pi^2 l}{t^2}$$

With the pendulum used

$$t = 1.040 \text{ seconds approximately}$$

$l = l_1 + l_2 + l_3$, where $l_1 = 104$ cm., $l_2 = 2.8$ cm., $l_3 = 0.2$ cm. approximately. With what precision must t and l be determined (assuming "equal effects" for these two components)?

How precisely must l_1 , l_2 , l_3 be found to obtain the required precision in their sum?

Indicate the number of significant figures to be carried in each part of the computation.

43. The volume of a sphere whose diameter is about 6 inches is desired within (1) a deviation of 0.45 cubic inches, (2) within 0.10 per cent. How closely must the diameter be measured in each case?

44. In measuring the ratio (h) of the arms of a balance, a weight of approximately 43 grams is weighed on each side.

$$h = \frac{\text{length right arm}}{\text{length left arm}} = \sqrt{\frac{w_1}{w_2}}$$

If this ratio is desired within 0.01 per cent., how precisely (in milligrams) must the weighings be made? How many significant figures should be used in the computations?

45. Using second order spectra from a grating spectrometer, the wave length is given by

$$\lambda = \frac{1}{2n} \sin \theta$$

If for a certain monochromatic light $\lambda = 0.50 \times 10^{-3}$ mm., approximately, and θ is found to be about 30° , how closely must θ be measured to give λ to 0.002×10^{-3} mm.? $n = 500$ and may be taken as exact.

46. The efficiency of a machine may be determined as follows:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Loss}}$$

Show that, if the error in the output measurement is to produce the same effect as the error in the loss measurement, the errors must be directly proportional to the quantities themselves.

47. In using a 10 meter slide-wire potentiometer the standard cell has a voltage of 1.01830 volt known within 0.00010 volt. The constant storage cell has approximately 2.2 volts and may be assumed connected across the terminals of the 10 meter slide-wire. It is desired to obtain the e.m.f. of an unknown cell within 0.00020 volt. The approximate value of the unknown is 1.5 volts. How much variation will be permissible in the settings of the sliding contact? Give answer in millimeters.

Could non-uniformity of the wire be neglected if such precision were required? Note that the same part of the slide-wire is used in both the reading for the standard cell and for the unknown. Under what conditions would the nonuniformity of the wire produce a minimum effect?

48. In using the resistance box direct reading potentiometer the total resistance is 22000.0 ohms. The standard cell has an e.m.f. of 1.01830 volts ± 0.00010 volt. The cell to be measured is approximately 1.5 volts and is desired within ± 0.00020 volt. What precision in ohms is required in the resistance determinations? Is this precision obtainable with ordinarily good resistance boxes provided the galvanometer is sensitive enough? Would a precision of 0.00020 volt in 1.5 volts be within the range of a laboratory experiment?

49. The resistance of a galvanometer is given by the equation

$$G = \frac{R_1 d_1 - R_2 d_2}{d_2 - d_1}$$

The errors in R_1 and R_2 are negligible. The value of the resistance G is desired within 1.0 per cent. Assuming equal effects for d_1 and d_2 , what precision in centimeters is required for d_1 and d_2 in each of the following cases?

Case A. d_1 = galvanometer deflection = 18 cms. approximately.
 d_2 = galvanometer deflection = 14 cms. approximately.
 R_1 = 300 ohms and R_2 = 400 ohms.

Case B. d_1 = 9 cms. and d_2 = 18 cms. approximately.
 R_1 = 50 ohms and R_2 = 0 ohms.

Is it possible with the d'Arsonval galvanometers used in the laboratory to read to such precision? The smallest division on the galvanometer scale is 1 mm.

50. The moment of inertia of a body is experimentally determined in terms of the following equation:

$$I = \frac{(F - f - ma)r}{\frac{a}{r}}$$

where F = 200,000 dynes (error negligible).

f = 10,000 dynes (approximately).

m = 200 grams (error negligible).

a = 30 cms./sec². (approximately).

r = 5 cms. within 0.10 per cent.

Suppose I is desired within 2.0 per cent. Let the error in a produce three times as great an effect as that of f . Find the actual deviations allowable in a and in f . Solve for I using the proper number of significant figures and express I correctly with its A.D.

51. Suppose four independent determinations of the Ice Point on the Absolute Fahrenheit scale gave

491.54° F ± 0.40° F.

491.63° F ± 0.20° F.

491.71° F ± 0.30° F.

491.64° F ± 0.12° F.

Compute the most probable value of the ice point *weighting* the individual determinations according to their respective precision measures, and compute the precision measure of the result.

52. The frequency of vibration of a stretched string is given by the formula

$$N = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

These quantities have the following approximate values:

N (the frequency) = 156 vibrations per second

l (the length) = 10 cm.

T (the total tension) = 98000 dynes
 and m (the mass per unit length) = 0.01 gm. per cm.

It is desired to obtain N within 0.50 per cent. How closely (according to "Equal Effects") must l , T and m be determined? If the uncertainty in N were due to errors in T alone, the other components l and m being measured so accurately as not to affect the result, how precisely would it be necessary to measure T in order that N might be dependable within 0.50 per cent.?

53. The volume of a vessel is to be obtained at a temperature $t = 100^\circ \text{C.}$ by measuring the volume of the vessel at 0°C. and using the equation $V_t = V_0(1 + 3\lambda t)$. The volume V_t is desired within 0.0050 cm^3 . If V_0 is 20 cm^3 approximately and λ is 0.000010 approximately, what precision will be required in V_0 , λ and t assuming them to have equal effects? What precision in V_0 to produce *three* times the effect of either λ or t , the effects of λ and t being equal?

54. The relation between the temperature centigrade and the e.m.f. of a platinum-rhodium thermo couple is given by the relation

$$\log t = \frac{\log \frac{E}{m}}{n}$$

If $E = 10.56 \pm 0.05$ millivolts, $n = 1.1850 \pm 0.0040$, and $m = 0.002676 \pm 0.000070$, what is the precision of t in degrees?

EXERCISE 1

VERNIER AND MICROMETER CALIPERS

Object.—(1) To study the principles of the vernier and the micrometer screw and their application in the vernier caliper and micrometer caliper, respectively. (2) Practice with these instruments in length measurements.

General Discussion.—The vast majority of the measurements made in a physical laboratory are ultimately measurements of distance. Two temperatures, for instance, may be compared by the difference in length of a thread of mercury; a pressure may be determined from the height of a barometer column or from the distance that the pointer of a pressure gauge moves; etc.

Measurements of length are always referred to divided scales, and when precision is desired, fractions of a division must be determined. One of three methods is employed for this purpose. The simplest is that of *estimation*, which depends upon the judgment of the observer. A second method, *more precise* and largely independent of judgment, is by means of the *vernier*, so named from Pierre Vernier, who invented the device in 1631. The greatest precision is attained through the application of the *micrometer-screw*.

THE VERNIER.—As already stated, the vernier is used to facilitate the reading of fractions of a division of a divided scale. It consists of a short auxiliary scale, movable along the main scale, the zero line of the vernier serving as the index of the main scale. This zero line is in general not at the end of this auxiliary scale, and the vernier does not rest directly against the object to be measured. This is indicated in Figs. 1 and 2. Verniers are generally so graduated that n divisions of the vernier equal in length $(n - 1)$ divisions of the scale. If, for example, a vernier is employed to determine tenths of a scale division, it may be constructed so that, while its total length is equal to *nine* divisions of the main scale, it is subdivided into ten equal parts, each vernier part, therefore, being equal in length to nine tenths of a scale division. Such a scale is shown in Fig. 1.

Suppose, as is represented in this sketch, that the index or zero vernier line lies between the main scale graduations corresponding to 1.4 and 1.5. The fraction of a division from 1.4 to the

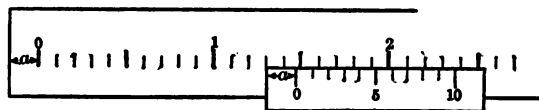


FIG. 1.

index line can be determined as follows: It is apparent that if, as is the case in the figure, line No. 7 of the vernier coincides with a line on the main scale, then vernier line No. 6 is $\frac{1}{10}$ of a scale division beyond the next lower scale line, vernier line No. 5 is $\frac{2}{10}$ of a scale division beyond its corresponding scale line, and so on. It follows, therefore, that the 0 line or index of the vernier is $\frac{7}{10}$ of a scale division beyond line No. 4 on the main scale, and the reading to the nearest tenth of a division is 1.47.

The following rule for reading by means of simple verniers is always applicable: *The vernier subdivides the smallest division of the scale into as many parts as the vernier is itself divided.* The reading consists of the sum of two parts; first, the reading of the main scale up to the line which immediately precedes the index or 0 of the vernier; second, the vernier reading. To facilitate the recording, express the fractional part of the main scale reading in the same denomination as the value of a single division of the *vernier*, then add to this the vernier reading obtained by noting the number of spaces between the index and that line on the vernier which coincides with a line of the main scale.

Sometimes, however, when it is desirable to read to the very end of the main scale, it is convenient to have the n vernier divisions equal in length to $(n + 1)$ of the scale. In this case the vernier and scale are numbered in opposite directions and the vernier is said to be "retrograde," but the above rule for reading still applies. It is obvious that in case the entire main scale shall be readable by means of a vernier, a combination of a "direct" and "retrograde" vernier may be used, in which case the above rule applies to either one.

All that has been said in regard to the straight vernier applies directly to the circular vernier used to measure angles. In this case, however, the vernier usually has 20, 30 or 60 divisions in-

stead of 10 or 25, and in reading it is customary to keep the results in terms of degrees, minutes, and seconds.

Applications.—Of the many instruments employing the vernier to facilitate the reading, the following may be mentioned: barometer, cathetometer, vernier caliper, spectrometer, and transit. The vernier caliper will be employed for practice in this exercise.

The Vernier Caliper.—For measuring quickly various small lengths with moderate precision, the vernier caliper is useful. It consists (Fig. 2) of a straight scale having two jaws whose plane and parallel surfaces are at right-angles to the length of the scale. One jaw is fixed and the other, movable along the

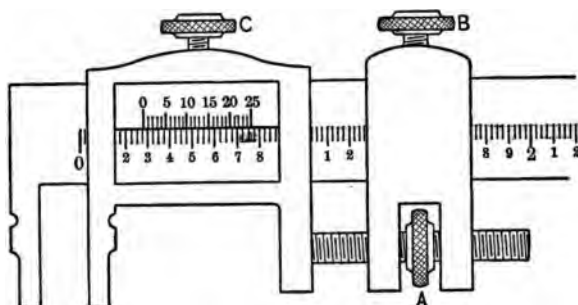


FIG. 2.

length of the scale or bar, carries a vernier the zero mark of which is so placed as to indicate on the scale directly the opening between the jaws. The object whose length is to be measured is placed between the jaws, which are adjusted until the object is *very lightly* held between them. The movable jaw is then clamped in position by set-screw (*C*), and the reading taken. The screws *A* and *B* are not used in such measurements. The bar is sometimes graduated on one side in inches, and on the other in centimeters. Every reading must be corrected for the *zero error* by subtracting, algebraically, the reading of the vernier when the jaws are closed.

A reading glass should be used in reading the vernier, and the instrument should be held facing a window or a light, if possible. It will often be found that the divisions of the two scales shift relatively to one another, according to the direction from which they are viewed. Care must be taken, therefore, to look down

on the instrument perpendicularly. This shifting with the point of view is called *parallax*, and must be guarded against whenever a vernier is used.

THE MICROMETER SCREW.—This device consists of a carefully made screw with a divided head which turns in an accurately fitting fixed nut. It is evident that the distance the end of the screw will move is proportional to the angle through which the head is turned. The number of complete turns is indicated by a scale on the instrument, while fractions of a turn are recorded by the divisions on the head of the screw. The pitch of the screw divided by the number of divisions on the head gives the distance that the end of the screw advances when the head is turned through one division.

This principle of the micrometer-screw is employed in the micrometer caliper, the spherometer, the dividing engine, the micrometer microscope, interferometer, theodolite, etc.

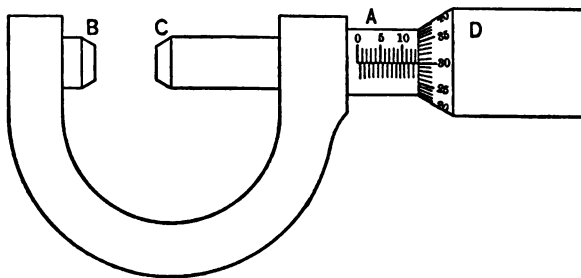


FIG. 3.

The Micrometer Caliper.—The micrometer caliper consists of a very accurate screw moving in a nut fixed to the frame of the instrument toward or from a stop called the anvil. The distance between the end of the spindle *C* and the anvil *B* (Fig. 3) can be determined in terms of the pitch of the screw, there being a longitudinal scale on the sleeve *A* to indicate whole turns of the screw, and divisions upon the thimble *D* to indicate fractions of a turn. In the standard instruments the screws have forty threads per inch, or twenty per centimeter. The sleeves of these instruments are divided into fortieths of an inch or into half millimeters, and the thimbles into twenty five and fifty parts respectively. Consequently, the instruments read, by the estimation of tenths or by the use of a vernier, which in some instru-

ments is marked upon the sleeve, to a ten-thousandth of an inch, or a ten-thousandth of a centimeter.

When using this instrument, place the object to be measured between the spindle *C* and the anvil *B*, and turn the screw down upon it until a very slight pressure is felt. *Never turn it down tight.* If the caliper is provided with a ratchet head for securing uniform pressure, it should be turned gently until the ratchet *slips one notch.* The readings on the sleeve *A* and the thimble *D* should then be recorded. The zero reading of the instrument, i. e., the reading when the spindle and anvil are in contact, should be made in the same way. This zero reading must be taken both before and after a series of measurements is made, and the average value should be applied as a correction to all readings. This quantity is known as the "Zero Error."

Apparatus.—A vernier caliper graduated to read to thousandths of an inch on one side and fiftieths of a millimeter on the other; two standard micrometer calipers, one graduated in inches and one in centimeters; and a ground cylinder whose dimensions are to be determined. The numbers or letters found on all the articles should be carefully recorded on the data sheet.

Manipulation.—(1) *Measurements of Length with Vernier Caliper.*—To determine the Zero Error [see important note, page 52] bring the jaws into contact and record the vernier readings in centimeters and in inches. Make two independent settings.

To measure the length of the cylinder the two set-screws (*B*) and (*C*) (Fig. 2) should be loose and the nut (*A*) should be approximately in the middle of the slow-motion screw. Place one end of the cylinder against the fixed jaw of the caliper and slide the movable jaw along the scale toward the cylinder until the latter is *just* held by the friction at the ends when the instrument is placed horizontally. Make nine such independent settings on the length of the cylinder and then two more independent zero settings of the caliper. Record reading in metric and English measure as indicated in Table I.

(2) *Measurements with Micrometer Calipers.*—In measuring the diameter of the ground cylinder with the metric and English standard micrometer calipers, the same number of readings should be taken with each instrument as in part (1), the student being careful to put a gentle and uniform pressure on the cylinder, and to estimate to one tenth of a division on the thimble, that is,

the vernier is a direct one and we wish to know the distance between the index and the zero of the scale in a negative direction, we must subtract the total number of vernier divisions from the vernier reading in order to obtain the zero with *proper sign*.

Computations.—Compute on the data sheet the mean value of each of the above four sets of measurements. In the fourth and eighth columns of each of the tables record the difference between this mean value and each of the original observations used in determining it. These differences are known as the deviations from the mean, and their mean, obtained without regard to sign, is known as the average deviation. This average deviation is an indication of the precision of the quantity measured, as has already been explained under the head of "Precision of Measurements."

Making use of the data, compute the volume of the cylinder in both metric and English measures. Divide the volume in cubic centimeters by the volume in cubic inches, extract the cube root, and indicate the significance of this result.

Compute the mass of the measured cylinder by taking its density as $8.57 \frac{\text{grams.}}{\text{cm.}^3}$

EXERCISE 2

THE SPHEROMETER

(Read Exercise 1 before performing this exercise)

Object.—Practice in the use of a spherometer to determine (1) the radius of curvature of the convex surface of a plano-convex lens and the focal length of this lens, (2) the thickness of a metal plate.

General Discussion.—A spherometer is an instrument made on the same principle, essentially, as a micrometer caliper. It consists primarily of a rigid metal tripod with three steel legs equidistant from one another and from a central screw which turns through a nut in the collar to which these three legs are attached, and moves at right-angles to the plane determined by the ends of these legs. This accurate micrometer screw terminates at its lower end in a rounded point, and has fastened to its upper end a disk graduated into 200 divisions and a milled head. A vertical scale on the instrument is used to determine the number of revolutions the screw has been turned from the zero position. If the pitch of the screw is known, this instrument affords a means of measuring with considerable precision small vertical distances both above and below the plane of the points of the legs.

Radius of Curvature of Lens.—By applying the spherometer first to a plane surface and then to the surface of a lens, the difference in the readings gives the height of a spherical segment determined by the three points of the legs. By Plane Geometry it may be shown that if a is the distance moved by the screw, and h the average distance between the legs of the spherometer and the point of the screw, then the radius of curvature of the surface is:

$$r = \frac{h^2}{2a} + \frac{a}{2}$$

Apparatus.—A spherometer, a true plane, a steel scale, a lens and a metal plate.

Manipulation.—(1) *Radius of Curvature.*—Place the instrument on the *plane* surface, being sure that the screw has been suffi-

ciently raised so that its point will clear the surface of the glass. Now, by means of the milled head, turn the spherometer screw down slowly, taking care that no greater pressure is exerted on one leg than on the others, until its point is in contact with the plane. As soon as the point touches the glass, the pressure due to the weight of the instrument is removed from the legs and the friction of the screw in the collar causes the whole instrument to revolve. Care must be taken in this operation not to exert unnecessary pressure on the top of the screw, as such pressure would tend to bend the tripod and thus produce an incorrect reading. This operation may be simplified and the action of the instrument made more definite by first locating, inside of a limit of about ten divisions, that position where the instrument will just *not* spin even if lightly tapped on one leg by the finger. By confining the attention to this region and slightly tapping the instrument in the direction in which the screw is being turned, it is possible to make very accurate readings.

When contact has been obtained, note the reading of the vertical scale and graduated disk, and record result by adding the disk reading expressed in centimeters to the vertical scale reading. Care must of course be taken to read the disk in the same direction in which the scale is read; i. e., readings on the disk should increase when readings on the vertical scale increase. Perform this operation nine times, using each time a *different* part (near the center) of the plane surface so as to obtain a good average value for the reading. In making a series of readings with the spherometer upon a given surface, it is best not to look at the disk reading until the setting has been made except in so far as to locate the approximate position. Otherwise it is difficult to avoid being influenced by a knowledge of previous readings. Each reading should be the result of an independent attempt to obtain an exact setting. Care should be taken in moving the spherometer not to injure the points nor to scratch the lens or the plate.

Now raise the screw so that its point will not be in contact with the lens surface when the instrument is placed upon it. Bring the screw into contact with the lens surface and take nine readings of the scale and disk. Using the mean of the nine readings on the plane surface and on the lens, compute the distance a in the formula by multiplying the difference in the means by the pitch of the screw.

To determine h adjust the point of the screw so that it is in the plane of the points of the legs and measure the distance from the screw to the end of the three legs by placing one of them and the point of the screw on the steel scale. The point of the screw should be placed on one of the principal lines and the position of the leg read to *hundredths of a centimeter*. Measure each distance *three* times, making *nine* readings in all.

(2) *Thickness of Plates*.—Adapt the principles discussed under manipulation to the measurement of the thickness of the metal plate.

Arrange data in the following form:

READINGS OF SPHEROMETER No. ...

On Plane		On Lens No.		On Plate No.		Distance h	
	d		d		d		d
.....							
.....							
.....							
.....							
.....							
.....							
.....							
.....							
.....							
.....							
.....							

Computation.—Compute the mean values and the a.d. and A.D. for each of the four sets of nine readings. Reduce the averages to centimeters and compute a in centimeters for the lens surface and also the thickness of the steel plate in centimeters.

From the values of a and h compute the radius of curvature of the convex lens surface. Finally compute by means of the following formula, in which $\mu = 1.520$ is the index of refraction of the glass, the focal length of the lens, using logarithms.

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{r_1} + \frac{1}{r_2} \right).$$

EXERCISE 3

THE MICROMETER MICROSCOPE

(Read Exercise 1 before performing this exercise)

Object.—The object of this exercise is to afford practice in the use of a Micrometer Microscope, (1) by determining its calibration constant, (2) by determining the distances between three rulings on a glass slide.

General Discussion.—A most excellent instrument for measuring small lengths with precision is the micrometer microscope, which is a combination of the micrometer screw and the compound microscope. The objective of the microscope produces an enlarged image of the object, which image should be brought, by focusing, into the plane of a system of movable spider lines. The position of the movable spider lines or cross-wires is controlled by a fine screw, the head of which is a drum usually divided into one hundred parts to facilitate reading fractions of a turn. Tenths of these divisions can be estimated. To aid in counting the number of whole turns there is in the field of view a saw-toothed scale. On some instruments each notch indicates one turn and every fifth notch is deeper than the others. In the instruments used in the laboratory one turn of the screw is indicated by the movement of the cross-wires from a point to a notch or vice versa. Every fifth point of this comb has a hole under it to indicate ten turns of the screw.

As the measurement of a length with this instrument consists in determining the exact number of turns of the screw required to move the cross-wires over the *magnified image* of the object to be measured, it is evident that the number expressing the length of this object will vary with the focal length of the objective and the length of the draw-tube. This fact makes necessary a factor called the "*calibration constant*." This constant is the distance *on the microscope stage* corresponding to one complete turn of the screw. The calibration constant is a *constant* only as long as the *same objective* is used at the *same distance* from the cross-wires.

Adjustment.—In using a micrometer microscope, first focus the *eye-piece* upon the cross-wires by grasping the micrometer

and moving the eye-piece in or out with a twisting motion while looking through the instrument which is directed toward a sheet of white paper. Next adjust the instrument so that its axis is perpendicular to the plane in which the object lies whose length is to be measured, and focus the microscope as a whole upon the object. The perfection of focus is indicated by the absence of parallax, that is, when the eye is moved sideways, the image remains stationary with respect to the cross-wires. If there is parallax the adjustments must be repeated. It is advisable at this point to call attention to the fact that the image given by a micrometer microscope is inverted.

Apparatus.—A micrometer microscope mounted on a suitable support; a stage micrometer (a standard scale cut on glass) which is graduated in millimeters, the last millimeter into tenths; a strip of glass with three lines ruled on it.

Manipulation.—Since the reading of the micrometer microscope differs in only one way from the reading of a simple micrometer caliper, when this point is thoroughly understood all difficulty in its use is obviated. This one difference is the following: The notches and points on the comb are not numbered as are the divisions on the sleeve of the micrometer caliper. For this reason we may ourselves number them as we please and call any convenient notch our zero notch. In order to reduce the chance for mistakes in counting it is always advisable to choose a point near the center as the arbitrary zero, as long as it is *outside* of the *image* to be measured, the image itself being as near the *center* of the field of view as possible. It is also advisable to have the zero on *that side of the center* which enables us to read the drum in an ascending direction when we make our readings.

In the micrometer caliper the object to be measured is held between the spindle and the anvil, and, assuming no zero error, one reading immediately gives the length of the object. In the micrometer microscope it is different. Here the image can be measured only by sighting on both ends. As long as both ends have to be independently sighted upon there is, first of all, no need to place one of these ends in a perfectly definite position, and even if this were attempted, it would not be correct to assume that the mere placing of one end would be as accurate as the mean value of n settings on the other. For this reason it is customary to set on both ends of the image, and the method

of measuring is then exactly similar to that of a distance measured with the central part of a rule, where both ends are read and the length (in inches, centimeters or millimeters) is obtained by subtracting one reading from the other. In the case of the micrometer microscope a length is obtained in a similar manner but is expressed in turns and fractions of a turn of the micrometer screw. Since the customary units of distance are the inch and centimeter, it becomes necessary to transform the length in *turns and fractions of a turn* into one of these units. The transformation factor is called the *calibration constant* of the instrument and is obtained in the following manner:

(1) *To Determine the Calibration Constant of the Instrument.*—Having focused the eye-piece on the cross-hairs, focus the microscope sharply on the stage micrometer, which should be placed on the base of the instrument. A sheet of white paper placed beneath the glass scale may make the graduations more readily visible. The graduated side of the glass plate should, of course, be uppermost, and the image of the graduations be parallel to the movable cross-hairs. Since it is best in the determination of the calibration constant to measure as large a known distance as possible, it becomes advisable to choose the arbitrary zero as far to one side of the center as practicable.

Next adjust the stage micrometer in such a way that one of the 0.1 centimeter lines is brought near this arbitrary zero. Turn the screw until the movable cross-hairs lie equidistant on each side of this line, being careful that the final motion of the screw is always in the same direction in order to eliminate errors due to lost motion or "back-lash." It is usually best to choose that direction which gives *increasing numbers* on the divided head of the screw. Read the whole number of turns from the arbitrary zero to the line by the notched scale and the fraction of a turn by the drum. Make nine independent settings in this way. Then turn the screw until the movable cross-hairs have traversed as many of the large divisions of the standard scale as are in the field of view and set them equidistant on either side of the last line. Read the notched scale and the divided head or drum. Make nine independent settings on this last line of the stage micrometer, also *record the distance on the stage micrometer thus measured*. Record all micrometer readings in tabular form similar to that suggested in Exercises 1 and 2.

(2) *To Measure the Distance Between the Rulings on a Glass Plate.*—Lay the glass plate on a mirror and focus the microscope upon the lines that mark out the distances to be measured. Be sure that the axis of the micrometer screw is parallel to the distance which is being measured.

Make nine settings on each of the three lines, recording in turns and fractions of a turn the distance of each from the *same* arbitrary zero notch. Nine consecutive readings may be taken on each line, or the lines may be read in rotation until the required number of readings has been obtained.

Computation.—Compute the mean of each of the five sets of observations. Obtain the best numerical estimate of the reliability of each mean.

Using the mean values of the settings on the two lines of the stage micrometer, compute the distance between them in turns of the screw. How precise do you estimate this value to be? Compute the calibration constant in centimeters per turn.

Knowing the calibration constant, compute the distance in centimeters between the first and second, also between the second and third lines on the glass plate.

EXERCISE 4

SIMPLE HARMONIC MOTION

PART I

Object.—To determine the acceleration of gravity by means of the Borda pendulum.

General Discussion.—For a simple pendulum, i. e., a mass concentrated at a point and supported by a weightless string, we have as the half-period t (or time of half a complete vibration)

$t = \pi \sqrt{\frac{l}{g}}$, where l is the length of the pendulum, i. e., the distance from the point of suspension to the center of oscillation, and g is the acceleration of gravity.

As the mass of the ball used in this exercise cannot be considered as concentrated at its center, the center of oscillation of the vibrating system actually lies a little below the center of the ball. The length of the equivalent simple pendulum may, however, be found as follows: For any compound pendulum the half-period is given by

$$t = \pi \sqrt{\frac{I}{Mgh}}$$

where I is the moment of inertia of the vibrating system about the axis of support, h the distance of its center of gravity below this axis, and M its mass. As the supporting wire is so light as to be negligible, h represents the distance from the axis of support to the *center* of the ball. The moment of inertia of the ball about any axis through its center is $I_0 = \frac{2}{5}Mr^2$ (Duff, A Text Book of Physics, p. 68, fourth edition), where r is its radius. The moment of inertia of the ball about the point of support is then (see Duff, p. 66)

$$I = I_0 + Mh^2 = \frac{2}{5}Mr^2 + Mh^2$$

Substituting this value of I in the above formula gives

$$t = \pi \sqrt{\frac{h + \frac{2}{5} \frac{r^2}{h}}{g}}.$$

A comparison of this expression with the formula for a simple pendulum shows the Borda's pendulum to be equivalent to a simple pendulum whose length is

$$l = h + \frac{2}{5} \cdot \frac{r^2}{h}$$

If l and t are known, g may be computed from the formula

$$g = \pi^2 \cdot \frac{l}{t^2}$$

Apparatus.—The apparatus consists of a Borda's pendulum, i. e., a metal ball suspended by a fine wire and arranged to swing on a hardened steel knife-edge; a telescope with cross-hairs; a telegraph sounder connected to a standard clock. The clock closes the circuit every second except the 59th, which is omitted for convenience in determining the beginning of the minute.

Manipulation.—The half-period t of the pendulum is to be determined by the "Method of Coincidences," by means of which a very accurate measurement is possible. First set the telescope at right angles to the path of the pendulum, letting the vertical cross-hair coincide with the image of the suspension wire when at rest. Start the pendulum, drawing the bob only a few centimeters to one side and being careful not to introduce any conical motion. Now set the sounder going and record the exact hour, minute and second when the sound is coincident with the passage of the wire across the cross-hair. If there is doubt as to which second the coincidence is best, make a record of the first one that might be taken and the last one; the true time of coincidence will be very approximately the average of the two recorded times. In this manner record consecutive coincidences for a period of at least 25 minutes.

It is evident that during each time interval between successive coincidences the Borda's pendulum makes either one more or one less half-oscillation than the clock pendulum, according as the former swings faster or slower than the latter. If, then, there are n of these coincidences during N seconds, the half period of the pendulum is $\left(\frac{N}{N \pm n} \right)$ multiplied by the rate of the clock.

Measure the length from the knife-edge to the bottom of the ball to the nearest tenth of a millimeter, and obtain the radius of the ball with a caliper.

Computations and Questions.—Compute the half-period of the pendulum, and the length of the equivalent simple pendulum. With these values of t and l compute g and determine the percentage deviation of this value from that given in the Smithsonian Tables for this locality. Take $\pi^2 = 9.8696$.

Are there any sources of error not corrected for? Which quantity, t or l , should be measured with the greater precision? By what per cent. will the calculations be affected if N be taken greater by 5 than the number used, n remaining the same? Can the correction $\frac{2}{5} \frac{r^2}{h}$ be neglected in computing l ? Give reason.

PART II

Object.—The study of Hooke's law and of linear simple harmonic motion.

General Discussion.—For the motion of any body to be simple harmonic it is necessary that its acceleration be toward the point of zero displacement and proportional to the displacement. The *period* of any simple harmonic motion is given by the formula

$$T = 2\pi \sqrt{\frac{-x}{a}}$$

where x = the displacement and a = the acceleration.

In the case of a stretched spring the restoring force F is, according to Hooke's law, proportional to the elongation of the spring. If a spring is, therefore, elongated an amount D by hanging on it a mass M , evidently $KD = Mg$, or $K = \frac{Mg}{D}$, where K is the constant for this spring. When this weighted spring is oscillating vertically, its motion satisfies the above condition for simple harmonic motion. Consequently, if the oscillating mass has a linear acceleration a for a vertical displacement x from the position of equilibrium, then

$$Ma = -Kx = -\frac{Mg}{D}x$$

whence

$$T = 2\pi \sqrt{\frac{D}{g}}$$

Apparatus.—The apparatus consists of a set of weights; a spiral spring; a scale for measuring the elongation of the spring; and a stop-watch or stop-clock.

Manipulation, Plots and Questions.—First determine the position of the lower end of the spring without load, using a mirror to avoid parallax in the readings. Put on a load of 20 grams and record the elongation produced. Increase the load successively by 20 grams each time until a load of 100 grams is reached; then increase by 50 grams each time up to about 300 grams, recording the elongation for each load. Remove the weights in order and record the position of the spring for decreasing loads. With these data plot a curve between the values of weight in grams and elongation to illustrate Hooke's law.

Compare the stop-clock with the standard clock in the laboratory and determine the error, if there be any, by letting it run for about two minutes. Determine the period of one *complete vertical oscillation* of the spring under loads of 50, 100, 150, 200, 250, and 300 grams. To do this, take with the stop-clock the time of 100 oscillations in each case.

With the above data plot a curve between the elongation and the square of the period. What conclusion may be drawn from the curve? Use some value of T^2 and D taken from the curve and solve for g . What is the percentage deviation of this value of g from the true value for this vicinity? Why is this method not so accurate as the simple pendulum for determining g ?

Other conditions being the same, would it be better in this experiment to use a more accurate method of timing, such as the method of coincidences? Give reasons.

EXERCISE 5

THE BEAM BALANCE

Object.—To become acquainted with the theory of the balance and to obtain practice in its use.

General Discussion.—For the accurate determination of the mass of a body in terms of a unit of mass, e. g., the gram, or of the weight of a body in vacuo, use is made of an equal-arm balance. This instrument is extensively used in scientific work and in many branches of industrial work where the accurate determination of the masses or of the weights of bodies is of prime importance. Balances of capacity 100 to 200 grams suitable for ordinary physical and analytical work read directly to 0.2 milligram and are sensitive to 0.1 milligram. Precision balances are sensitive to 0.05 mg. Assay balances are often sensitive to 0.002 mg. See Catalogues: Christian Becker Inc., also Volland and Sons.

Construction.—The equal-arm balance consists of a beam with a knife-edge at the center and one at each end, the beam being often of a truss design in order to give maximum rigidity with minimum weight and made of a hard rolled aluminum alloy. It may be lowered by means of a suitable device until the central knife-edge swings on a plane. The pans of the balance are hung from the knife-edges by planes and thus remain suspended in a vertical position with the swinging of the balance. In high grade balances the knife-edges and planes are of agate, though steel ones are used in the cheaper instruments. A long pointer attached to the beam swings in front of a fixed scale, the deflections being read directly or by a telescope. A small adjustable mass on the pointer or a mass on the beam above the central knife-edge serves to raise or lower the center of gravity of the balance-beam. At one end or at both ends of the beam will be found a small nut to make the effective weights of the arms of the balance equal, and thus cause the pointer to swing equally on both sides of the middle division of the scale. The balance is mounted in a glass case provided with a door which must be kept closed during all measurements. Pan-arrests are usually pro-

vided to guard against injury to the knife-edges when lowering the beam and pans and to prevent the pans from swinging. The arrests are controlled by a button on the front of the balance-case, and a milled head is provided to raise the beam and pans off their knife-edges. There is usually a level or a plumb bob attached to the balance-case, adjustable by the levelling screws on the case.

Reduced to its elements an equal-arm balance consists of a rigid beam supported horizontally at its center on a knife-edge and loaded at its ends. The center of gravity of the balance must be below the central knife-edge since, if it were above this point, the beam would be in unstable equilibrium, and if the center of gravity were at this point, the equilibrium would be neutral.

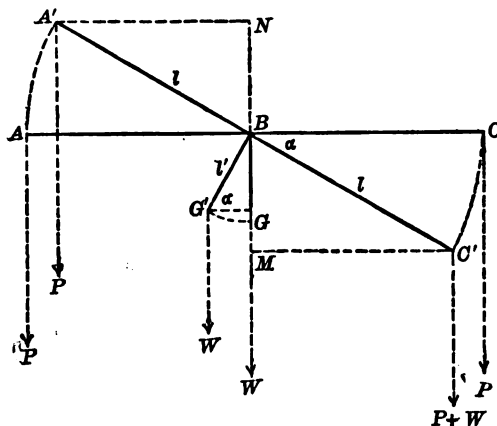


FIG. 4.

Sensitiveness.—The sensitivity or sensitiveness of a balance is usually defined as the angle through which the beam will swing for 1 milligram difference in the weight on the two pans. A simple relation may be found for the angle α through which the balance turns for a given difference in weight, w , in the two pans, for the case where the three knife-edges lie in the same straight line. If W equals the weight of the beam, P weight of pan with load, l the half-length of the beam or the distance between the central knife-edge and either end one, l' the distance of the center of gravity of the balance below the central knife-edge, then by reference to Fig. 4,

$$\tan \alpha = \frac{wl}{Wl'} \quad (1)$$

From this equation it follows that:

1. The longer the beam, the greater the sensitiveness.
2. The lighter the beam, the greater the sensitiveness.
3. The smaller the distance between the point of suspension of the balance and the center of gravity, the greater the sensitiveness.

These conditions for sensitiveness are somewhat incompatible, since an increase in the length of the balance-arms would probably necessitate an increase in the weight W of the beam to maintain the required rigidity. Again an increase in the length of the balance-arms or a decrease in the value of l' , while increasing the sensitiveness, will make the time of swing of the balance excessive. Treating the balance as a compound pendulum, it can be shown that its time of swing (half-period) is

$$t = \pi \sqrt{\frac{Wk^2 + 2Pl^2}{Wl'g}}$$

where k = radius of gyration of beam and pointer about the axis of swing, g = acceleration due to gravity and W, P, l, l' have the same significance as before. The period of a balance should not exceed 15 seconds. Short-beamed balances constructed with light beams have a half-period of 6 to 10 seconds, thus saving time in the weighing in comparison with long-beamed balances, whose time of swing is usually from 10 to 15 seconds.

As a matter of convenience the sensitiveness should be independent of the load. This is possible only when the knife-edges lie in the same straight line, and since this condition can be fulfilled only for one load on account of the bending of the beam, the best balances are constructed to produce it for a mean load. Hence there is usually a slight increase in sensitiveness with increasing load up to the mean load, followed by a decrease.

Weights.—Measurements with a balance presuppose a correct set of masses, called commonly a set of weights, or else a set that has been carefully calibrated. For ordinary purposes the weights from 1 gram to the highest are of lacquered brass, the fractional weights, from 500 to 20 milligrams being of nickel silver or german silver, and from 20 milligrams to 1 milligram of aluminum. The highest grade of weights are of platinum, or of gold-plated brass, the units smaller than 1 gram being of platinum to 20 mg. and from 20 milligrams to 1 milligram of aluminum. Ordinary weights furnished by reputable makers cannot be relied upon to

better than one milligram, and it becomes necessary in all work of precision to make a careful calibration of the weights before undertaking any measurements. The *correctness* of a set of weights should never be taken for granted, for they are often in error to an extent that affects the accuracy of the weighings to a larger degree than some of the errors for which corrections are usually made, such as the inequality of the balance-arms and the buoyancy effect of the air. For the calibration of a set of weights, see Kohlrausch, *Physical Measurements*, page 40, or Goodwin, *Physical Laboratory Experiments*, page 45.

Procedure in Weighing.—In using the balance it is customary to place the body whose mass or weight is desired in the left-hand or object-pan, and in the middle of the right-hand or weight-pan such masses (so-called weights) as are estimated sufficient to counterbalance the body. If, when the beam arrestment is partially lowered, the pointer moves toward the object, then the weights are too great and a smaller weight should be substituted for one on the pan and the beam arrestment lowered again. The final adjustment is made with the smallest weights and the rider, both beam and pan arrestment being released after each change.

The Rider.—In order to weigh objects to the necessary degree of refinement, one is obliged in many cases to record the mass to 0.1 milligram or less. This gives rise to at least two difficulties. First, it is impractical to work with standard masses as light as a milligram; and second, when the balance is being used to this degree of precision, the currents of air produced on opening and closing the case to increase or decrease the mass would materially affect its equilibrium. These difficulties are admirably overcome by a little device called a "rider." This consists of a piece of platinum or aluminum wire bent into such a form that it will readily ride on the beam of the balance. The right-hand beam (in some balances both beams) of the balance is divided from a point above the central knife-edge to a point above the right pan knife-edge into twelve equal parts, of which the first and last are not indicated on account of the construction of the balance. It is obvious by the principle of moments that a mass placed at the point ten, for instance, could have an apparent weight only 10/12 of what it would weigh if put in the pan. If, therefore, a rider is constructed which weighs 12 mg., it will have

the same effect when placed on division 1 as 1 mg. on the right-hand pan, and so on. These 12 divisions on the beam are divided into halves, fifths, twelfths, etc., according to the sensitiveness of the balance. The rider may be moved along the beam by means of a finger on a sliding rod which is manipulated from outside the case, thus making it unnecessary to open the case after an adjustment within a few milligrams of a balance has been obtained. In some balances the beam is divided on one side or on both sides of the central knife-edge into either 50 or 100 divisions, the weight of the rider being properly chosen.

In the Chainomatic Balance made by Christian Becker, Inc., the rider is replaced by a small gold chain suspended at one end from an adjustable screw fitted to the beam of the balance and, at the other, from a hook held by a sliding block. The latter may be moved vertically on a flat rod that is graduated in milligrams. The block, which is marked as a vernier for obtaining readings to 0.1 milligram, is moved along its supporting rod by a vertical spiral drive in the rear, which in turn is rotated by a thumb screw through bevel gears. The device is very convenient and is a time-saver, especially when weighings are not desired closer than one milligram. The method of swings may be used with this device the same as with the ordinary rider.

Precautions in the Use of a Balance.—During the process of weighing it is necessary to observe several precautions. The most important of these, together with certain general instructions, are given below:

1. The balance case should stand on a pier or support as free from mechanical vibration as possible.

2. If the balance case cannot be maintained at a constant temperature, it must at least not be exposed to unequal heating. Direct sunlight falling on the balance will cause irregularities in the weighings.

3. A dish of unslacked lime or other drying material should be kept in the balance case to prevent rusting of the knife-edge, if of steel, and to eliminate as much as possible the influence of moisture during weighings.

4. A balance is designed to carry a maximum load that should not be exceeded.

5. To guard against injury to the knife-edges from jarring, the balance beam must always be raised off the knife-edges when balance is not in use.

6. The beam must always be lowered slowly and carefully on to its knife-edge. When the balance is in perfect adjustment and the beam unloaded, the pointer should be in equilibrium at the center of the scale. If this is not the case, this adjustment may be secured by moving the small nut attached to the end of the balance-arm. *The student, however, should never alter this nor any other adjustment of the balance*, but should bring any apparent maladjustment to the attention of an instructor.

7. The release of the pan-arrests will usually cause the beam to vibrate as desired. If not, fan one of the pans gently with a motion of the hand. *Never touch the pan with the forceps, nor attempt to set the beam swinging by suddenly lowering it upon its knife-edge.* The pans should have no swinging motion.

8. Avoid parallax in reading by having the line of sight perpendicular to the scale. A mirror may be placed back of the scale for this purpose.

9. Arrest the beam *only* when it is passing through its position of equilibrium.

10. *The beam must never rest upon the knife-edge while objects or standard masses are being added to or removed from the pans.*

11. The weights (standard masses) should be handled *only* with the pincers provided. The large weights should be placed near the center of the pan, and all weights used should be arranged on the pan in the order of their denominations.

12. The final adjustment of the rider and all final weighings should always be made with the balance-case shut.

13. On the completion of a weighing, record carefully the mass of the object. To guard against the incorrect recording of the mass, check up the weights on the scale-pan by noting the missing weights in the box (in which everyone should have its own place), and then add up the weights as they are returned to the box.

14. When the weighing is finished, remove the weights from the pans, lift the rider from the beam, and close the balance-case.

15. Any substance that would injure the scale pans should be weighed in a suitable vessel.

16. If possible, substances should be near the temperature of the room when weighed, otherwise serious errors in the weighings may result from the air currents that will be created.

Methods of Weighing.—I. *Method of Equal Swings.*—For ordinary weighings the common method is to adjust the balance

under *zero load* so that the pointer swings equal distances on both sides of the middle scale division, allowance being made for the diminution in amplitude of successive swings resulting from friction at the knife-edges, air resistance, etc. The body to be weighed is then placed on the left scale-pan and weights are added to the right-hand pan until by means of the rider the pointer swings once again approximately equally on both sides of the middle division. This method is only applicable to balances provided with riders, and it is found not to recommend itself when the extreme precision attainable with the balance is desired.

II. *Method of Swings*.—It is as a general rule better to make a measurement of two quantities depend on how much they differ rather than on their equality. This is the principle used in the Method of Swings, which has the additional advantages over the Method of *Equal Swings* (1) that it is the only method applicable to balances without riders, (2) it does not require the balance to be adjusted under no load so that the pointer will swing equal distances to the right and left of the middle scale division, (3) the reading of the pointer while swinging takes account of the decay of oscillations due to friction and the resistance of the air.

In determining the *zero-point*,—that is the position of rest of the pointer under no load,—it must be remembered that the balance oscillates about the central knife-edge like a compound pendulum. The oscillations are damped by friction at the bearings and by air resistance, and, therefore, each swing will be a trifle shorter than the preceding one. The final position or rest-point will not, then, be exactly half-way between a turning-point on the left and the next turning-point on the right, but more nearly half-way between the average of two successive turning-points on the left and the intervening one on the right. To secure a higher degree of precision, record five successive turning-points on the left and four on the right. Take the average of each set, and the rest-point is halfway between these two average values. For convenience consider the ivory scale numbered from left to right with the central line numbered ten, the first long line on the left being zero and the last one on the right being twenty. A redetermination of the zero-point should be made at frequent intervals, and, if any alteration is found to occur, the mean of the positions before and after the weighings should be used.

Following the determination of the zero-point, the body to be weighed should be placed on the left scale-pan and the pointer brought nearly to the zero position (within one or two scale divisions) by placing weights in the other scale-pan and the rider on a whole milligram division. The rest-point under this load should be found. Next, a small weight should be added to or subtracted from the right side of the balance by shifting the rider over 1 or 2 milligram divisions so that the rest-point falls within 1 or 2 scale divisions of the zero point but on the side *opposite* the position previously determined. The rest-point under these conditions should be determined. Evidently the change of 1 or 2 milligram has shifted the rest-point from one side to the other of the zero-point. Since, for small deflections, the change in the rest-point is proportional to the unbalanced force which produces it, it is possible to compute by proportion what fraction of a milligram would be required in the right pan to bring the pointer to the *zero-point*, i. e., to balance exactly the body. This quantity added to or subtracted from the weights on the right side of the balance gives the weight of the body.

The following example will make this clear:

Suppose the following data have been obtained.

Zero point.....	9.90
Point of equilibrium for 13.9710 gm.....	10.33
Point of equilibrium for 13.9720 gm.....	9.12

Then the mass required to balance the body is

$$13.9710 + (0.0010) \frac{10.33 - 9.90}{10.33 - 9.12} = 13.9714 \text{ gm.}$$

Sources of Error in the Use of a Balance.—In all the weighings there are three principal sources of error to be considered.

1. Errors due to inequality in length of the balance-arms.
2. Errors due to buoyancy of the air.
3. Errors in the standard masses employed.

1. CORRECTION FOR INEQUALITY OF BALANCE-ARMS.—The error arising from the inequality in the length of the balance-arms may be corrected by determining the ratio of the lengths of the arms. Since, however, this ratio may vary with the load on account of difference in rigidity of the arms, this method is seldom employed. It is more customary to use the method of Borda or of Gauss.

Borda's Method (Method of Taring, or Substitution).—The body to be weighed is balanced by adding to the other pan shot, fine sand or other material. The body is then taken away and standard masses put in its place until the former reading of the balance is obtained. The sum of the masses put on the pan gives the mass of the body whatever the relative length of the balance-arms.

Method of Gauss (Double Weighing).—This method consists simply in weighing the body first in one pan and then in the other. Thus, if a body of true mass M when placed in the right-hand pan balances mass A , and when placed in the left-hand pan balances mass B , then, calling R and L the lengths of the respective arms, it can be shown that

$$MR = AL$$

$$ML = BR$$

therefore

$$M^2 = AB$$

$$M = \sqrt{AB}$$

Thus the true mass is the geometrical mean of the apparent masses. Since, as a rule, A and B are very nearly equal, the *arithmetical mean* may be used in most cases.

The method of Gauss has several advantages over that of Borda. Its precision is greater, for it can be shown that the probable error of Borda's method is decidedly greater than that of Gauss. Furthermore, less time is taken, and there is the additional advantage that one weighing furnishes a check on the other.

2. CORRECTION FOR THE BUOYANCY OF THE AIR.—For the accurate comparison of masses it is essential, when the weighing is made in air, that their densities be the same, as otherwise an error will be introduced. The reason for this is that the body and the standard masses being of unequal volumes will displace different amounts of air and hence will be buoyed up by different forces (Archimedes' principle). Consequently, the mass of a body as found by an equal-arm balance in air, after all the corrections for instrumental errors have been made, will only be the apparent mass under the conditions of temperature and pressure at the time of the weighing. The correction for buoyancy is not always applied, but in accurate weighing it becomes generally

of sufficient importance to be considered, especially when there is a decided difference of density between the body weighed and the standard masses used.

This correction may be deduced in the following way: Let V , M and Δ denote the volume, mass, and density of the body, while v , m , δ have similar significations with respect to the standard masses used. These quantities are so related that

$$V = \frac{M}{\Delta} \quad \text{and} \quad v = \frac{m}{\delta}$$

Now every body when weighed in air is buoyed up by an amount equal to the weight of the volume of air which it displaces. It follows, therefore, that the buoyant effect on the body is λVg and that on the standard masses is λvg where λ is the density of the air (about 0.0012 gram per cm^3 under the average laboratory conditions). From this it follows that, when m is adjusted to give equilibrium in an equal arm balance,

$$Mg - \lambda Vg = mg - \lambda vg$$

For convenience, this expression may be written as follows:

$$M - \lambda \frac{M}{\Delta} = m - \lambda \frac{m}{\delta}$$

Since M is nearly equal to m , the following approximate formula, which is much more convenient, can be used without appreciable error.

$$M = m + \lambda \left(\frac{m}{\Delta} - \frac{m}{\delta} \right)$$

In the weighings made in this course it will be sufficient to use two significant figures for λ , Δ and δ , and consequently also for m , in the correction term.

Manipulation.—This exercise consists (1) in determining by the Method of Swings the position of equilibrium of the pointer of the balance with *zero load*,—the *zero-point*; (2) in finding the weight of an object by the Gauss method, using the Method of Swings, correction being made for the buoyancy of the air.

(1) Allow the beam of the balance to rest on the knife-edge by turning the milled-head knob on the front of the case. Release the pan-arrests, if any, and set the balance swinging by fanning one of the scale-pans very gently with the hand, so that the

pointer moves over not more than six or seven divisions. Close the balance-case. Note nine successive turning-points of the pointer, *five* on one side and *four* on the other, reading to the tenth of a scale division. Determine the zero-point from these readings. In order that an error resulting from viewing the pointer from different directions as it swings (error of parallax) may not be made, a small mirror may be placed immediately back of the pointer. Then as the pointer swings, the eye may be moved so that the pointer always covers its reflection in the mirror. After the reading of the swings has once begun, do not raise the front of the case until the nine readings have been taken. Take care in starting the swing not to set the scale-pans moving in any way other than up and down with the knife-edges from which they hang.

Make two additional series of observations, first with an *amplitude* of swing of *four* or *five* divisions to either side of the center, and second with an *amplitude* of *eight* or *nine* divisions. Average the results of the three series to obtain the *zero-point*.

(2) For this part of the exercise an object is provided whose true mass is to be found by the Method of Swings, correcting for inequality of balance-arms by the method of Gauss, using the zero-point already obtained and determining the new rest-point by two sets of nine readings each as in part (1). In the calculations the density of the air λ may be assumed to be 0.0012 gram per cm^3 under the ordinary atmospheric conditions prevailing in the laboratory, and the density of the standard masses δ may be taken as 8.5, while the density of the object should be obtained from an instructor. The final result should be expressed to 0.0001 gram.

On the completion of the exercise, lower the balance-arm, arrest the pans, return the weights to their proper places in the box, and close the balance-case.

Problem.—Compute the error that would be introduced in the determination of the mass of exactly 1 gram in air of each of the following substances through neglect to apply the buoyancy correction, and compute the true mass in each case. Assume the weights to be brass.

Substance	Density
cork	0.2
ether	0.72
water	1.00
nickel	8.9
platinum	21.5

EXERCISE 6

SPECIFIC GRAVITY OF LIQUIDS.—PICNOMETER

Object.—To determine to 0.1 per cent. the specific gravity of a liquid at 30° C. referred (1) to water at 30° and (2) to water at 4° C. by means of a Sprengel-Ostwald picnometer.

General Discussion.—The *density* of a body is its mass per unit volume. If m be the mass of a homogeneous body and v its volume, its density will be expressed as follows:

$$\Delta = \frac{m}{v}$$

Since the volume of a body varies with the temperature, its density will have a slightly different value at different temperatures. Therefore, in stating the density of a body, its temperature should also be given. The numerical value of the density must depend on the system of units chosen.

The *specific gravity* or relative density of a substance is the ratio of its mass to the mass of an equal volume of some substance at a definite temperature taken as a standard; also since the quantity of matter in a body is proportional to its weight, we may write

$$S = \frac{m}{m'} = \frac{w}{w'}$$

where m and m' are the masses of equal volumes of the substance and of the standard respectively, and w and w' their corresponding weights. From the definition it follows that the specific gravity is also equal to the ratio of the densities, i. e.,

$$S = \frac{\Delta}{\Delta'}$$

For solids and liquids, water at its temperature of maximum density, 4° C., is chosen as the standard. In the case of gases and vapors, either air or hydrogen under normal pressure and temperature of 0° C. is the standard used.

From the definition it is evident that the value of the specific gravity is independent of the system of units used.

The specific gravity of a liquid is often referred to water at the same temperature as the liquid, but to avoid ambiguity the temperature both of the substance and of the water to which it is referred should always be stated in giving the value of the specific gravity: thus $S_{30/4}$; $S_{30/30}$.

It is in general an easy matter to determine experimentally the ratio of the mass of the body to the mass of an equal volume of water at the same temperature, but it is not easy nor convenient to keep the water at its temperature of maximum density throughout the experiment. Since, however, the density of the water at different temperatures is known, it is a simple matter to reduce the value of the specific gravity referred to water at any temperature t to water at 4°C .

If m_1 = apparent mass of the vessel filled with liquid at $t_1^\circ \text{C}$.

m_2 = apparent mass of the vessel filled with water at $t_2^\circ \text{C}$.

and f = mass of vessel empty,

then the approximate specific gravity of the liquid at t_1° referred to water at t_2° is

$$S = \frac{m_1 - f}{m_2 - f} \quad (1)$$

This value may differ from the specific gravity referred to water at 4°C . by a per cent. or more owing to the following causes:

(a) Change of volume of the vessel due to its change in temperature from t_2° to t_1° .

(b) Buoyancy of the air.

(c) The fact that the water is at t_2° instead of being at its temperature of maximum density 4°C .

When the specific gravity referred to water at t_2° is desired to 0.1 per cent., it may be necessary to correct the value of S in equation (1) for errors arising from causes (a) and (b).

Correction for Expansion of Vessel.—This correction will be eliminated in this exercise by filling the picnometer, both with the liquid and with the water, at the same temperature, i. e., by making $t_1^\circ = t_2^\circ$.

Correction for Buoyancy of the Air.—If M' and M'' be the true mass in vacuo of equal volumes of the liquid and of the water respectively at the same temperature t_1° , and if m' is written for $(m_1 - f)$ and m'' for $(m_2 - f)$, then assuming that the balance

has equal arms,

$$M' - \frac{M'}{\Delta} \lambda = m' - \frac{m'}{\delta} \lambda$$

$$M'' - \frac{M''}{D_{t_1}} \lambda = m'' - \frac{m''}{\delta} \lambda$$

where λ (approximately 0.0012) stands for the mass of 1 c.c. of air under the conditions of temperature, pressure and humidity at the time of weighing, δ stands for the density of the weights, Δ for the density of the liquid at t_1° and D_{t_1} for the density of water at t_1° . (See Exercise 5.)

Dividing one equation by the other gives

$$\frac{M' \left(1 - \frac{\lambda}{\Delta}\right)}{M'' \left(1 - \frac{\lambda}{D_{t_1}}\right)} = \frac{m' \left(1 - \frac{\lambda}{\delta}\right)}{m'' \left(1 - \frac{\lambda}{\delta}\right)}$$

$$\frac{M'}{M''} = \frac{m'}{m''} \cdot \frac{\left(1 - \frac{\lambda}{D_{t_1}}\right)}{\left(1 - \frac{\lambda}{\Delta}\right)} = \frac{m'}{m''} \cdot \frac{(D_{t_1} - \lambda)}{(\Delta - \lambda)} \cdot \frac{\Delta}{D_{t_1}}$$

Now

$$\frac{M'}{\Delta} = \frac{M''}{D_{t_1}}$$

hence

$$1 = \frac{m'}{m''} \cdot \frac{(D_{t_1} - \lambda)}{(\Delta - \lambda)}$$

and

$$\Delta = \frac{m'}{m''} \cdot (D_{t_1} - \lambda) + \lambda \quad (2)$$

Therefore the specific gravity of the liquid at t_1° referred to water at the same temperature is

$$S_{t_1/t_1} = \frac{\Delta}{D_{t_1}} = \frac{(m_1 - f)}{(m_2 - f)} \frac{(D_{t_1} - \lambda)}{(D_{t_1})} + \frac{\lambda}{D_{t_1}}$$

$$= \frac{m_1 - f}{m_2 - f} + \frac{\lambda}{D_{t_1}} \left(1 - \frac{m_1 - f}{m_2 - f}\right) \quad (3)$$

Correction for the Density of the Water.—Since the density of the water at 4° C. is unity, the specific gravity of the substance

at t_1° referred to water at 4° C. becomes

$$\begin{aligned} S_{t_1/4} &= \frac{\Delta}{1} = \frac{m_1 - f}{m_2 - f}(D_{t_1} - \lambda) + \lambda \\ &= \frac{m_1 - f}{m_2 - f}D_{t_1} + \lambda \left(1 - \frac{m_1 - f}{m_2 - f}\right) \end{aligned} \quad (4)$$

Apparatus.—The Sprengel-Ostwald picnometer for the determination of the specific gravity of liquids to 0.1 per cent. is shown in Fig. 5. It is so constructed that when filled with water or with a liquid the adjustment may be readily made at the same temperature by suspending the entire instrument, except its capillary ends, in a bath kept at constant temperature. This eliminates the correction for expansion. The capacities of picnometers vary from 10 c.c. to 50 c.c. according to the accuracy of the results desired. The one to be used in this exercise has a capacity of approximately 10 c.c.

Manipulation.—Thoroughly clean and dry the picnometer. In cleaning use first ordinary tap water and finally rinse with distilled water. The drying process may be hastened by blowing warm air through the picnometer while it is held in a bath of boiling water. During this process the drops of water which form at the end of the capillary tube should be removed with clean filter paper. When the picnometer is dry, allow it to cool to *room* temperature, suspend it by the attached wire from a hook on the end of the balance-arm and weigh. Weighings need not be made on both sides of balance.

All weighings in this exercise should be made to 0.1 milligram, using the method of vibrations described in Exercise 5. The scale readings obtained in the determination of the zero point and the points of equilibrium should all be recorded.

Next fill the picnometer with distilled water. This is most readily accomplished by drawing in the water at *A* by sucking through a rubber tube connected to the end *C*.

Suspend the picnometer up to the capillary tubes in the water bath. This bath must be stirred continuously and its temperature maintained at 30° C. by *adding* from time to time as much *warm water* as necessary. After about five minutes bring the meniscus of the distilled water to the reference mark *B* (see Fig. 5) by carefully touching the point *C* with a bit of filter paper.

If, when the tube is filled to the point *C*, the meniscus remains stationary at the mark *B*, the picnometer and contents have assumed the temperature of the bath. If not, a little distilled water may be added by touching a drop on the end of the stirring rod to the capillary end *C*. During this adjustment the *picnometer must not be removed from the bath*.

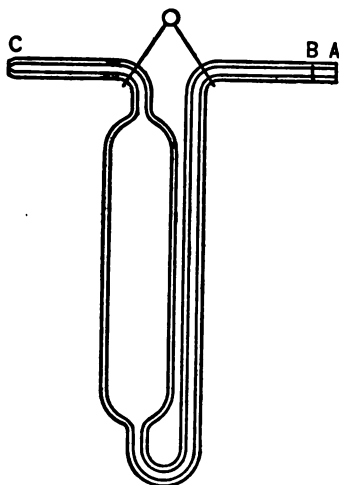


FIG. 5.

When the final adjustment has been made, dry the picnometer and weigh. Always hold the picnometer by the attached wires so that pressure of the hand on the sides shall not cause the liquid to overflow.

Following exactly the above procedure, find the weight of the picnometer when filled with the liquid at 30° C. whose specific gravity is desired. Then rinse out the picnometer.

Computations.—With these data compute the specific gravity of the liquid referred (1) to water at 30° C. and (2) to water at 4° C. *Before making this com-*

putation, determine by use of formula (3) or (4) how much the buoyancy correction affects the result. If the buoyant effect is 0.03 per cent. or less, the correction is negligible and should be omitted. The calculation for determining whether the correction is negligible should be carried out only to two significant figures.

Problem.—If the picnometer is filled with a liquid at t_1° and with water at t_2° , the approximate formula for specific gravity is

$$S_{t_1/t_2} = \frac{m_1 - f}{m_2 - f}$$

Neglecting the correction for the buoyancy of the air, *derive* the formula for S_{t_1/t_2} , making the correction for change in the volume of the picnometer due to a change of temperature from t_2° to t_1° . (Mean coefficient of linear expansion of glass referred to centigrade scale $\alpha = 0.0000086$.)

EXERCISE 7

SPECIFIC GRAVITY OF SOLIDS.—SPECIFIC GRAVITY BOTTLE

Object.—To determine to 0.1 per cent. the specific gravity of a solid at 30° C. referred to water at 4° C. by means of a specific gravity flask.

General Discussion.—To determine the specific gravity of any substance it is necessary to find the weight of a certain volume of that substance and divide it by the weight of an equal volume of water. These values can be obtained by three weighings provided the solid is insoluble in water. Let

m = apparent mass in grams of the substance dry

m_2 = apparent mass of bottle plus water at t° C.

m_3 = apparent mass of bottle plus water plus substance at t° C.

Then the approximate mass of the water displaced by the substance is $(m + m_2 - m_3)$.

The approximate specific gravity at t° C. is then

$$S = \frac{m}{m + m_2 - m_3}$$

This expression when corrected for air buoyancy and referred to water at 4° C. gives, according to equation (4) of Exercise 6,

$$S_{4/4} = \frac{m}{m + m_2 - m_3} D_t + \lambda \left(1 - \frac{m}{m + m_2 - m_3} \right) \quad (1)$$

where D_t is the density of water at t° C. in grams per c.c. and λ is the density of air.

Apparatus.—The apparatus consists of a sensitive beam balance, a specific gravity flask and the substance of unknown specific gravity. There are in addition a water bath, a thermometer and stirrer, and a supply of warm water to secure any desired temperature. The flask is in general a small glass flask of about 50 c.c. capacity. The stopper is carefully ground to fit the neck and perforated with a small hole. When the flask is filled with water and the stopper inserted, the excess liquid passes

out through the hole in the stopper, thus giving a very exact adjustment to the same volume each time the flask is filled.

Manipulation and Computations.—Dry the sample of the unknown and weigh. *All weighings should be made to 0.1 milligram. Use the method of vibrations and record all readings, but weigh only on one side.* (See Exercise 5.) Clean and rinse the specific gravity flask. Next fill it with distilled water at room temperature, making sure that no air bubbles are present. If any do appear, cause a large bubble to travel along the inside of the bottle so that it may coalesce with and remove the smaller bubbles. When the bottle has been thus filled, insert the glass stopper firmly but not tightly. Place the flask in the water bath, which by continuous stirring and the addition of warm water is kept at 30° C. The bath should be just deep enough to come to the bottom of the stopper. When the water in the flask escapes, owing to the unequal expansion of glass and water, it should be removed with filter paper. After water ceases to escape, carefully dry the flask and weigh.

Next wet the specimen and insert it into the flask. Fill the flask with distilled water and remove air bubbles. Place the flask in the 30° C. bath and allow it to come to this temperature. Remove the flask, carefully dry on the outside and weigh.

Care should always be taken in handling the flask not to exert sufficient pressure by the hand to force water out of the stopper.

Having made the three necessary weighings, calculate the specific gravity of the substance ($S_{30/4}$) by means of formula (1). Throughout the computations use the proper number of significant figures to insure a precision of 0.1 per cent.

EXERCISE 8

YOUNG'S MODULUS BY STRETCHING

(Read Exercise 3 before preparing this exercise)

Object.—(1) To verify Hooke's Law and (2) to determine Young's modulus for a metal in the form of a wire.

General Discussion.—Read Duff, pages 109–115, and 118–120 (fourth edition).

In the theory of elasticity a *change* in the size or *the shape* or *both* of a body is called a *strain*. When the change is one of volume only, it is called a volume strain and is measured by the ratio of the change in volume to the original volume. When the change is one of shape without change of volume, the strain is called a shear. In the case of the elongation or compression of a rod or wire, the strain is a linear one, and is measured by the ratio of the change in length to the original length.

The force per unit area within the body tending to restore it to its original size and shape is called the *stress*. From the discussion in Duff, it is evident that up to the elastic limit the stress may equal numerically the ratio of the *applied force* to the area on which it acts.

Theory and experiment show that for small strains the ratio of the *stresses* to the *corresponding strains* is constant for a given body. This is Hooke's law. The ratio of stress to strain is called the coefficient of elasticity of the material.

$$\frac{\text{stress}}{\text{strain}} = \text{constant} = \text{coefficient of elasticity} \quad (1)$$

For isotropic bodies there are as many coefficients of elasticity as there are different kinds of strains. For example, there is a coefficient of volume elasticity or bulk modulus, and a coefficient of rigidity. The former is characteristic of all forms of matter, the latter of solids. An important coefficient of elasticity for solids is Young's modulus or stretch modulus.

Young's Modulus.—If a rod or wire of a given length, cross-section and material is acted upon by a stretching force, the resulting elongation is proportional to the force applied. The

ratio of stress to strain in this case is Young's modulus for the material, i. e.,

$$\text{Young's Modulus} = Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{\text{area}}}{\frac{l}{L}} = \frac{F}{\pi r^2} \cdot \frac{L}{l} \quad (2)$$

in which F is the force stretching the wire, L the original length of the wire, r its radius, and l the elongation. When the values of the quantities on the right-hand side of this equation have been determined experimentally and expressed in proper units, this *stretch coefficient* of elasticity or Young's modulus may be computed.

Approximate numerical values of Young's modulus or stretch modulus.

	Y	Y
Copper wire.....	12×10^{11} dynes/cm. ²	17×10^6 lbs./in. ²
Glass.....	6.5×10^{11} dynes/cm. ²	9×10^6 lbs./in.
Iron (cast).....	12×10^{11} dynes/cm. ²	17×10^6 lbs./in.
Iron (wrought).....	20×10^{11} dynes/cm. ²	29×10^6 lbs./in. ²
Steel (Bessemer).....	22×10^{11} dynes/cm. ²	31×10^6 lbs./in.
Wood (Oak).....	1.0×10^{11} dynes/cm. ²	1.4×10^6 lbs./in. ²
(Poplar).....	0.5×10^{11} dynes/cm. ²	0.7×10^6 lbs./in. ²

Apparatus.—Two long wires of the given substance (spring brass in this case) are hung from a fixed support on the wall. This arrangement has the advantage of avoiding any error due to changes of temperature in the room which would change the length of a wire under test; it also avoids any possible error due to the yielding of the supporting clamp. Masses are permanently attached to the bottom of each wire to support the stretching weights and to keep the wire straight. Four iron weights are provided to produce the stretching force. These wires hang in front of a micrometer microscope by which the elongation is measured.

Manipulation and Computations.—(1) *Verification of Hooke's Law for a Material in the Form of a Wire.*—The statement of Hooke's law for tension (see equation 2) is $\frac{F}{\pi r^2} \cdot \frac{L}{l} = \text{constant}$. Since $\frac{L}{\pi r^2}$ will remain practically constant, Hooke's law may be

verified by showing experimentally that $\frac{F}{l}$ is a constant. The forces F , which are the weights of the iron masses used to produce the stretching, may for this purpose be expressed in grams. The elongation l is measured by the micrometer microscope, the elongation being the *change in distance* between two light cuts on indicators attached to the suspended wires.

Focus the eye-piece of the micrometer sharply upon the cross-wires. See that the microscope is so arranged that the addition of the maximum load (4 weights) to the longer wire, the *one to be tested*, does not throw the image of the corresponding cut out of the field of view. Choose a convenient point near the bottom of the field of view on the notched scale as the zero reading of the microscope, and use this scale for recording the number of turns of the micrometer-screw.

First read the position of the cut on each wire with reference to the zero of the microscope when the fixed weights only are attached to the wires. The difference of the readings on the two cuts measures the distance between them. Make two independent determinations of this distance. Next place one weight on the support attached to the longer wire, and after four minutes make four independent determinations of the distance between the two cuts. Continue by adding the second, third and fourth weights, determining the distance between cuts by four independent settings for each load. Make similar determinations as the weights are removed one by one.

Note.—After each load wait four minutes before beginning the new set of readings.

The distances between the cuts can be changed to centimeters by the use of the calibration constant of the micrometer microscope. This can be obtained from an instructor. Determine the elongation corresponding to each load on the test wire, using in each case the average of the four measurements of the distance between the cuts.

Compute the ratio $\frac{F}{l}$ for each elongation observed, where F is the total force in *grams* (exclusive of the permanent weight) producing the elongation l .

(2) *To Determine Young's Modulus.*—The height of the support above the mark on the indicator has been determined and

can be obtained from the instructor. This is the length of the wire under test. Measure the diameter of the wire under test with a micrometer caliper nine times at as many different places between these points as can be reached. Then, after reducing the average value of $\frac{F}{l}$ obtained from Part (1) to dynes per cm. of elongation, compute the value of Y using the formula

$$Y = \frac{L}{\pi r^2} \cdot \frac{F}{l} \text{ dynes per cm.}^2$$

Throughout the entire experiment be careful to observe the rules regarding significant figures.

Problems.—(1) Write out the dimensions of longitudinal stress, longitudinal strain, Young's modulus.

(2) Calculate, using the above data, Young's modulus for spring brass wire in pounds per square inch and in kilograms per square millimeter.

(3) Compute the deviation measure of the average value of F/l .

EXERCISE 9

COEFFICIENT OF RIGIDITY BY TORSION

(Before performing this experiment review the general discussion of Exercise 8)

Object.—(1) To verify Hooke's law for torsion. (2) To prove that the angle of twist of a rod varies directly as the length and inversely as the fourth power of the radius. (3) To determine the coefficient of rigidity.

General Discussion.—Hooke's law, which is fundamental in the theory of elasticity, states that stress is directly proportional to strain. The types of stress and strain in a twisted rod are shearing stresses and shearing strains.

A change of shape without any change of volume is called a shearing strain, which, if small, may be measured by the linear displacement between two parallel planes in the body which are at a unit distance apart. The tangential force per unit area applied to one of these planes in order to produce the given strain is called the stress. The coefficient of rigidity n is the ratio of the shearing stress to the shearing strain. (For a more extended discussion of this subject the student is referred to Duff, fourth edition, pp. 111–114.) In the case of a twisted rod it is evident that the shearing strain is not the same at all points of a plane perpendicular to its axis, the strain at any point being proportional to the distance of that point from the axis of the rod. All points on a circle whose center lies on the axis of the rod will suffer the same shearing strain.

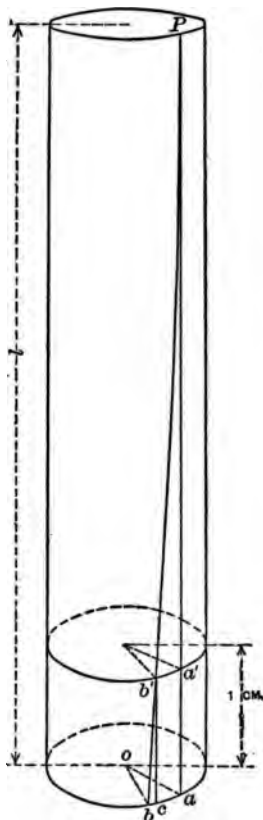


FIG. 6.

The rod under test may be looked upon as made up of a number of thin-walled concentric cylindrical tubes. Let the radius of any such tube be r , its thickness dr and length l , and let one end of this tube be fixed while the other end is twisted through an angle θ . In Fig. 6 Pa is a line on the cylinder parallel to the axis before the torque is applied. Pb is the new position of this line after the torque is applied. The angle aob is called the angle of twist or θ , and the twist per centimeter length is $\frac{\theta}{l}$.

Since the strain is a pure shear, the relative *linear* displacement of two cross-sections which are unit distance apart, i. e., the shearing strain, is equal to arc (ab) — arc $(a'b')$ = arc (cb) = $\frac{r\theta}{l}$.

This linear distortion produces at all points of any cross-section of the tube a restoring force which is in the plane of the cross-section and perpendicular to the radius of the tube. The magnitude of the force per unit of cross-sectional area, i. e., the shearing *stress*, is, by Hooke's law, n times the strain, or $\frac{nr\theta}{l}$ dynes per cm.² where n is the shear modulus. As the stress has different directions at different points of the section it will be found to produce no resultant force. It does, however, produce a *torque* about the axis of the tube which is equal to the (stress $\times$ area of the cross-section $\times$ radius of the tube). Or the torque which one portion of the tube exerts upon the adjacent portion is

$$\frac{(nr\theta)}{l} \times (2\pi r dr) \times (r) = \frac{2\pi n\theta}{l} r^3 dr.$$

The torque between contiguous portions of the tube is the same at all cross-sections, and hence the torque which must be applied at the end of the tube to maintain it in the twisted condition is $\frac{2\pi n\theta}{l} r^3 dr$. Considering the solid rod as made up of such tubes, it follows that the torque required to twist the rod through an angle θ is the sum of the torques required to twist the tubes of which it is made up. If the radius of the rod is R , this sum will evidently be

$$\text{Torque} = \int_0^R \frac{2\pi n\theta}{l} r^3 dr = \frac{\pi n\theta R^4}{2l}$$

If this torque is produced by an external force F having a moment-arm h , we have

$$Fh = \frac{\pi n \theta R^4}{2l}$$

or

$$n = \frac{2Fhl}{\pi R^4 \theta} \quad (1)$$

An inspection of this equation discloses certain important relationships:

First: For a given value of l and r the ratio $\frac{Fh}{\theta}$ is a constant.

This constant is numerically equal to the amount of torque required to twist the rod through one radian and is technically called the *constant of torsion*.

Second: The angle of twist θ resulting from the application of a given torque Fh is related to the length and radius as follows:

$$\theta \propto \frac{l}{r^4}$$

For an application of the relation $\theta \propto \frac{l}{r^4}$ to quartz fibers, see Watson, A Text-Book of Practical Physics.

Apparatus.—Two wires of the same material and of the same length (about one meter) but of different diameters are to be tested. These are held in a horizontal position, being rigidly fastened at one end to a heavy iron upright and at the other end clamped firmly to a torsion head graduated in degrees. Weights of 200 grams each acting at the circumference of the head are the applied forces. Ball bearings render the friction error negligible. Meter sticks and calipers are provided for the necessary length and diameter measurements.

Manipulation.—(1) Clamp the larger wire in position, adjusting its length to 100 cm. See that the band is smooth and free from kinks. Take two readings of the zero position of the torsion head. Slightly tap the wheel and allow it to come to rest before each reading. Observe the readings of the angle of twist to a tenth of a degree as three 200 gram-weights are *successively* added to the pan and then as they are removed. Take a pair of readings for each load, waiting two minutes between changes of load. Finally take two more readings of the zero position.

Since the radius $\frac{d}{2}$ enters the equation to the fourth power, the diameter of the wire under test must be determined with considerable care. Take nine readings of the diameter with micrometer calipers at various points along the wire and use the mean as the best representative value. The arm h of the acting torque is the radius of the torsion wheel, which should be determined from four measurements of its diameter.

Repeat the experiment with 100 cm. and then with 50 cm. of the smaller wire. This completes the experimental part of the exercise.

Computation.—(1) According to Hooke's law the angular twist at the end of the rod under torsion is proportional to the applied torque, that is, $\frac{Fh}{\theta}$ is a constant. Any deviations will be due

to experimental errors. As h is a constant in this exercise, $\frac{F}{\theta}$ should be constant, or, evidently, θ per 200 gram-weight should be constant.

Using the method shown in the footnote compute for both wires the different values of θ (expressed in degrees) per 200 grams.

(2) From equation (1), it follows that the angle of twist of the rod per 200 gram-weight should vary directly as the length and inversely as the fourth power of the radius, i. e.,

$$\theta_{\text{mean}} \propto \frac{l}{r^4}$$

To compare the theoretical with experimental results, compute the following relations:

(a) With r constant compare the ratio of the lengths of the small wire with the experimental relation

$$\frac{\theta_{\text{mean}} \text{ for } 1/2 \text{ small wire}}{\theta_{\text{mean}} \text{ for small wire}} = \dots\dots\dots$$

and compute the per cent. difference.

(b) With l constant compare the relations:

$$\begin{aligned} \frac{r^4 \text{ for small wire}}{r^4 \text{ for large wire}} &= \text{-----}, \\ \frac{\theta_{\text{mean}} \text{ for large wire}}{\theta_{\text{mean}} \text{ for small wire}} &= \text{-----}; \\ \text{per cent. difference} &= \text{-----}. \end{aligned}$$

(3) Compute by equation (1) the coefficient of rigidity n in *dynes per square centimeter* for each of the three tests made. (See footnote for the method of computing the best value for θ , the mean angle of twist per 200 gram-weight.) Note that in the equation θ must be expressed in radians.

Arrange all data and results neatly in suitable tabular form.

(4) *Problem.*—Compute the value of n when expressed in kilograms per square millimeter and in pounds per square inch.

Note.—The following method should be used in computing θ , the *mean* angle of twist per 200 gram-weight: If the total twist

due to 200 grams is, say 1.47°	$= 1.47^\circ$	per 200 grams
due to 400 grams is, say 2.89°	$= 1.445^\circ$	per 200 grams
due to 600 grams is, say 4.34°	$= 1.447^\circ$	per 200 grams
due to 400 grams is, say 2.91°	$= 1.455^\circ$	per 200 grams
due to 200 grams is, say 1.44°	$= 1.44^\circ$	per 200 grams
<u>1800</u>	<u>13.05°</u>	

then, if the sums equal 1800 grams and 13.05° , the most probable value of θ per 200 gram-weight which can be obtained from this set of readings is $13.05 \div 9 = 1.450^\circ$. This method gives to each observation precisely the amount of consideration which it deserves. Thus it gives a weight of 3 to the observed displacement of 600 grams and a weight of 2 to the observed displacement of 400 grams. Why is this true?

EXERCISE 10

TORSION PENDULUM

(Read general discussion of Exercise 9 before performing this exercise)

Object.—(1) To determine with a torsion pendulum the moments of inertia of a number of bodies about an axis through the center of gravity. (2) To determine with the same apparatus the coefficient of rigidity of a material in the form of a wire.

General Discussion.—(1) When a force is applied to a rigid body capable of rotation about an axis, rotation will result unless the line of action of the force passes through this axis. The effect of a force to cause rotation is measured by the product of the force and the perpendicular distance h from its line of action to this axis. This product is known as the *moment* of the force or the *torque*. The angular acceleration A that results depends not merely upon the magnitude of the torque Fh but also upon the *moment of inertia* I of the body about the given axis in accordance with the relation

$$Fh = IA \quad (1)$$

In this exercise a rigid body is fastened to the lower end of a metal wire. The upper end is clamped so as to allow an angular displacement of the system about the wire as an axis, thus forming a torsion pendulum. If a torque Fh be applied to the rigid body, producing a twist in the wire, the internal stresses developed within the wire produce an equal and opposite torque tending to restore the system to its original position of equilibrium. Experiment shows that the magnitude of the internal stresses increases with the angular displacement, and the torque which they produce is proportional to the angle of twist at the end of the wire and opposite in sense. Therefore

$$Fh \propto -\theta \quad \text{or} \quad Fh = -k\theta \quad (2)$$

where k is a constant for any given wire. Combining equations (1) and (2), we have

$$A = -\frac{k}{I}\theta \quad (3)$$

This relation states that the angular acceleration in any position is proportional to the angular displacement for that position, but in opposite sense, which is the condition for Angular Harmonic Motion. (Duff, pages 82-84 (fourth edition).) Therefore, it follows that, if the torsion pendulum is twisted and released, there will result isochronous oscillations of period

$$T = 2\pi \sqrt{-\frac{\text{angular displacement}}{\text{angular acceleration}}} = 2\pi \sqrt{-\frac{\theta}{A}} \quad (4)$$

Whence from equation (3)

$$T = 2\pi \sqrt{\frac{I}{k}} \quad (5)$$

This very important relation furnishes a method for finding *experimentally* moments of inertia, for, if two bodies be successively fastened to the end of the same torsion wire and set into oscillation, the periods will be to each other directly as the square roots of the moments of inertia, i. e.,

$$T_1 : T_2 = \sqrt{I_1} : \sqrt{I_2} \quad (6)$$

(2) Again, theory as well as experiment shows that if r is the radius of the wire, l its length and n the coefficient of rigidity, then

$$Fh = \frac{\pi n \theta r^4}{2l}$$

where Fh is the applied torque. Whence from equation (2)

$$k = \frac{Fh}{\theta} = \frac{\pi n r^4}{2l}$$

Substituting this value for k in equation (5), and transposing, we have

$$n = \frac{8\pi I l}{T^2 r^4} \quad (7)$$

Equation (7) offers a means of determining experimentally the *rigidity modulus* n of the *material* of which the wire is made.

Apparatus.—A circular disk of known moment of inertia fastened to a steel wire about 175 cm. long, thus constituting a torsion pendulum; a wall-bracket and clamp to hold the pendu-

lum firmly at the upper end; a cast-iron hollow cylinder of the *same mass as the disk*; a stop-clock; a counter; and suitable devices for measuring the required lengths and diameters.

Regular homogeneous bodies have been selected for this exercise since the moment of inertia of each may be computed from its dimensions. The following formulas ($I = \Sigma mr^2$) may be readily deduced by the aid of the integral calculus:

The moment of inertia of a homogeneous *circular disk* of radius R and mass M about its shortest axis of symmetry is

$$I = \frac{MR^2}{2} \quad (8)$$

The moment of inertia of a hollow circular cylinder about its axis of generation is

$$I = M \frac{(R^2 + r^2)}{2} \quad (9)$$

where R and r are the exterior and interior radii, respectively.

Manipulation and Computation.—(1) The object of this part of the exercise is to determine *experimentally* the moment of inertia of the *hollow cylinder*, whose mass is the same as that of the *circular disk*. The result is then to be compared with its value as computed by equation (9), using the measured dimensions.

A radial chalk line about an inch in length should be made on the upper surface of the oscillating body in such a position that it is hidden from the observer by the wire when the pendulum is at rest. This will assist in the counting of the oscillations. Set the pendulum into torsional oscillation about the axis of the wire by displacing it through an angle of about 20° . To prevent any other motion steady the wire near the base with the fingers. The period of the torsion pendulum must be determined with considerable care to ensure even moderate precision in the experimental values of I . For this reason the student is required to obtain in each case *three independent values* for the time of half-period by observing the *exact number of times that the chalk mark passes behind the wire from either side in an interval of about eight minutes* (the time being recorded to the nearest $\frac{1}{5}$ second). A registering device is provided to reduce the liability to error. The student should be careful to note direction of swing when counting. The stop-clock which is used to record the time should

be checked against the laboratory clock for eight minutes, and, if in error, the necessary correction applied. In the final computations the half-period used must be the average of the three independent values. These independent values should check to within 0.1 per cent. The period T is then twice this average value.

Determine in the above manner the period T_d of the torsion pendulum when vibrating with the circular disk only. Determine next the period T_{d+c} of the torsion pendulum when the hollow cylinder is added to the disk. Then calling the moment of inertia of the hollow cylinder I_c , compute its value by the formula

$$I_c = I_d \frac{T_{d+c}^2 - T_d^2}{T_d^2} \quad (10)$$

using the known moment of inertia of the disk I_d as computed by equation (8).

The measurements of the dimensions necessary to compute by equations (8) and (9) the moment of inertia of the disk and the cylinder should be made to 0.01 cm. Each dimension should be measured four times and the average used in computing I . Record the mass in grams stamped on these bodies.

Assuming the correct value of I_c to be that found by the use of equation (9), compute the percentage error in the experimental value found by the method of equation (10).

Note.—The student should note carefully that the moments of inertia of these two bodies of the same mass and the same external dimensions are not the same but vary with the distribution of the mass about the axis of rotation.

(2) To calculate the coefficient of rigidity n of the torsion wire in dynes per square centimeter: Determine the length of the torsion pendulum, i. e., of the portion of the wire that is twisted, using two one-meter scales. To obtain this length proceed as follows: Make four independent measurements of the distance between the lower face of the clamp (two settings in front and two behind the wire) and the upper face of the disk. Then to the mean of these measurements add the thickness of the disk minus the diameter of the wire. The reason for this procedure will be clear upon inspection of the apparatus. Measure the diameter of the wire with a micrometer caliper to the full pre-

cision of the instrument. The value used should be the mean of at least nine readings taken at different points along the wire. Using the value I_d computed for the disk and the corresponding observed value T_d , compute n by means of the following equation

$$n = \frac{8\pi I_d l}{T_d^2 r^4}$$

Arrange all work in a neat and orderly manner.

Problem.—Verify correctness of equation (10).

EXERCISE 11

RADIUS OF GYRATION AND MOMENT OF INERTIA OF A BODY BY SWINGING IT AS A GRAVITY PENDULUM

Object.—The determination of the radius of gyration and moment of inertia of a body by swinging it as a compound pendulum about a horizontal axis.

General Discussion.—The student is asked to review the subject of moment of inertia and the compound pendulum by referring to his textbook. (Duff, fourth edition, §§ 81, 82, 85, 89, 90, 92 and 120.) The following equations will then be clear, in which I_0 is the moment of inertia about an axis through the center of mass, I is the moment of inertia about any axis *parallel* to the one through the center of mass, k_0 and k are the corresponding radii of gyration, h is the perpendicular distance between the two axes, M is the mass of the rotating body.

$$I = Mk^2 \quad (1)$$

$$k^2 = k_0^2 + h^2 \quad (2)$$

$$I = I_0 + Mh^2 \quad (3)$$

Furthermore, if l is the length of a *simple pendulum* having a half-period t equal to that of the body oscillating as a compound pendulum, then,

$$l = \frac{g}{\pi^2} \cdot t^2 \quad (4)$$

also

$$l = \frac{I}{Mh} = \frac{k^2}{h} \quad (5)$$

From which we get

$$I = lMh \quad (6)$$

$$k^2 = \frac{I}{M} \quad (7)$$

Apparatus.—A connecting rod; stop-watch; Fairbank's scale; meter stick; and knife-edges.

Manipulation and Calculations.—The first part of this exercise consists in the experimental determination of the moment of inertia and the radius of gyration of a connecting rod, Fig. 7, about two parallel axes through the points *A* and *B* perpendicular to the plane of the paper. The second part consists in computing the value of these quantities about a parallel axis through the *center of mass* of the body using the data obtained in Part I.

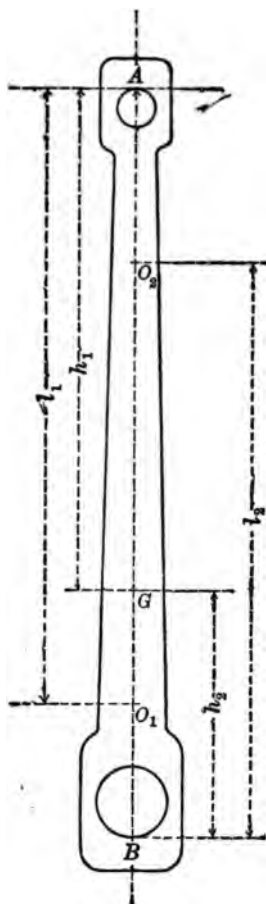


FIG. 7.

PART I.—A section of one of the connecting rods used in this exercise is shown in the figure. Let *G* be its center of mass, and *O*₁ and *O*₂ the centers of oscillation when the connecting rod is suspended from points *A* and *B*, respectively. Let *h*₁ and *h*₂ be the distances of the center of mass from points *A* and *B*, respectively.

From equations (6) and (7) we have about an axis through *A*

$$I_1 = l_1 M h_1 \quad \text{and} \quad k_1^2 = \frac{I_1}{M}$$

and about an axis through *B*

$$I_2 = l_2 M h_2 \quad \text{and} \quad k_2^2 = \frac{I_2}{M}$$

*To Determine the Lengths *l*₁ and *l*₂.*—Swing the pendulum about *A* and obtain the time of a single swing (half-period). Substitute this value in equation (4) and compute *l*₁, assuming *g* to be 980

cm. per sec. per sec. Then suspend the rod from *B* and find *l*₂. The measurement of the period must be made with considerable care to ensure even moderate precision in the value of *I* and *K*. To secure this precision set the rod oscillating about axis *A* and with a stop-watch determine the time required for 100 single swings. In the same way determine the time for 200 and for

300 swings. Observations should be made only when the pendulum is in its mean position, i. e., in the middle of the swing, and the first passage through this position should be called 0, the next 1, etc. The stop-watch should be checked against the laboratory clock for ten minutes, and if in error the necessary correction must be made.

A similar set of observations should be made when the rod is oscillating about the axis B .

If the three observations do not indicate a discrepancy of more than one swing, the period should be determined by adding up the total number of swings and the total time. This method will give the proper weight to the three sets of observations.

To find the quantities M , h_1 and h_2 , support the rod *horizontally* with the point A resting on a knife-edge on a platform balance and with point B on an edge not on the balance. Record balance reading. Then reverse and weigh again. By the principles of statics, if W_1 and W_2 are the respective components of the weight W of the bar, then

$$W = W_1 + W_2$$

$$h_1 = \frac{W_2}{W} (h_1 + h_2)$$

$$h_2 = \frac{W_1}{W} (h_1 + h_2)$$

Obviously $(h_1 + h_2)$ equals the distance between points A and B . With the values of l_1 , l_2 , h_1 , h_2 and M compute the values of I_1 , I_2 , k_1 and k_2 in c.g.s. units by substituting in equations (6) and (7).

PART II.—This part of the exercise consists in computing the moment of inertia and radius of gyration of the body about an axis through the center of mass parallel to the axis through A as follows:

With the values k_1 and h_1 , k_2 and h_2 , already found, compute the radius of gyration k_0 , using equation (2). Take the average of these two results as the best value of k_0 .

With this value of k_0 compute the moment of inertia I_0 , using equation (1).

EXERCISE 12

MOMENT OF INERTIA BY MEASUREMENT OF TORQUE AND ANGULAR ACCELERATION

Object.—(1) To review briefly linear and angular accelerated motions. (2) To determine the moment of inertia of a circular disk and of a rectangular bar about an axis through the center of mass.

General Discussion.—To cause a rigid body to move with a uniformly accelerated translatory motion, a constant resultant force must be applied whose line of action passes through the center of mass. Similarly a constant resultant torque must be applied to produce in a rigid body a constant *angular acceleration*. The magnitude of this angular acceleration A about a given axis depends on the magnitude of the torque L and on the moment of inertia I of the rotating body about this axis, in accordance with the relation

$$L = Fr = IA$$

where the torque L is due to a resultant force F applied at the end of an arm r .

In this exercise a mass m is suspended by a piece of flat tape of negligible mass. The tape passes vertically up over a light pulley of radius r_1 and then horizontally to a drum of radius r capable of rotation about a vertical axis. The moment of inertia of the light pulley is so small that it may be neglected without appreciable effect on the precision. The frictional torque L_1 at the bearing of the pulley and L at the axis of the drum may be assumed constant and independent of the speeds arising in this exercise.

Bearing in mind the fundamental relation in mechanics that

$$\text{sum of the actions} = \text{sum of the reactions},$$

the equation of motion of the mass m is given by

$$mg - T_1 = ma \tag{1}$$

where T_1 is the pull of the tape on the mass m .

Since there is a frictional torque L_1 tending to oppose the rotation of the pulley, the tension T above the pulley must be less than T_1 , and, therefore,

$$(T_1 - T) r_1 = L_1 \quad (2)$$

The equation of motion of the drum with attached masses, whose moment of inertia about the vertical drum axis is I , is

$$Tr = IA \frac{a}{r} L \quad (3)$$

By elimination of T_1 , equations (1), (2) and (3) reduce to

$$IA = (mg - f - ma) r \quad (4)$$

where

$$f = \frac{L_1}{r_1} + \frac{L}{r}$$

is a constant having the dimensions of a force acting with a lever arm r producing a torque equivalent to the frictional reactions at the bearings.

Since the linear acceleration of the falling mass m and the tangential linear acceleration of a point on the circumference of the drum are equal, equation (4) may be written

$$I = \frac{(mg - f - ma) r}{\frac{a}{r}} \quad (5)$$

an equation that may be employed for the experimental determination of I .

This same quantity $I = \Sigma mr^2$ may be computed in the case of certain bodies of regular shape whose density is constant or uniformly varying. Thus the moment of inertia of a homogeneous circular disk of mass M and radius R about an axis through its center of mass and perpendicular to the plane of the disk is equal to $1/2 MR^2$. For a parallelepiped of length L and breadth B , the moment of inertia about an axis through its center of mass and normal to the plane LB is $1/12 M(L^2 + B^2)$. (See Duff, pages 62-68, fourth edition).

Apparatus.—The apparatus consists of a light hard rubber drum free to rotate about a vertical axis; bodies of unknown moment of inertia which can be firmly fastened to the rubber

EXERCISE 13

IMPACT

Object.—(1) To demonstrate the conservation of linear momentum by the impact of two elastic bodies. (2) To find the coefficient of restitution. (3) To compare the kinetic energies of the two bodies before and after impact.

General Discussion.—In order to study central impact use is made of suspended bodies whose centers of mass move in the same path and for which the point of contact is also in this path. This method is employed because of the ease with which the velocities may be found and the fact that the motion may be considered one of pure translation without friction. When one suspended body strikes another, the whole impulse may be used in giving to the system a motion of translation in the direction in which the first body was moving, or part of the impulse may be used in setting up wobbling motions. If the impact takes place at a point called the center of percussion such motions are not produced. The center of percussion is at a distance from the axis of suspension equal to the length of the equivalent simple pendulum, and when the masses of the supporting cords are small compared with the mass of the spherical objects used and their length large compared with the radius of the spheres, the lower end of this equivalent simple pendulum is very near the center of mass of the spherical object.

By the Second Law of Motion any change in linear momentum is the result of the action of a force; if the only forces acting are those due to impact, then, since by the Third Law of Motion action and reaction are equal and opposite, the quantity of motion (regard being given to the algebraic sign) is unchanged by impact. In other words, the sum of the momenta before impact is equal to the sum of the momenta after impact. Let us represent the larger moving mass by M and the smaller mass (at rest) by m ; then, if U represents the velocity of M before impact, V its velocity after and v the velocity imparted to m , it follows from the statement above that

$$MU = mv + MV \quad (1)$$

If m has an initial velocity, u , precisely the same line of reasoning gives

$$MU + mu = mv + MV \quad (2)$$

This is the principle of the Conservation of Linear Momentum.

(2) *Perfectly elastic* bodies after collision would separate with the same velocity relative to each other as that with which they approach; *inelastic* bodies would not separate at all. The actions of actual bodies lie between these two extremes, and the ratio of the *velocity of separation* to the *velocity of approach* is called the *coefficient of restitution*. Expressing this symbolically,

$$e = \frac{v - V}{U - u}$$

From this it follows that for perfectly elastic bodies e would equal unity, for perfectly inelastic bodies e would equal zero. For actual materials e is a fraction, less than unity, whose value varies with the material.

The fact of imperfect restitution is not contrary to the conservation of Linear Momentum nor to the Third Law of Motion. It may diminish the *action*, but the *reaction* is diminished by the same amount, always remaining, however, equal to the action; thus there is no unbalanced force, and hence the change in linear *momentum* of the system is zero.

(3) The usual result of imperfect restitution is a change in the *kinetic energy* of the system at the time of impact, the change in the amount of the energy causing vibration, change of shape, sound and heat. Read Duff, 4th edition, §§ 174 and 176.

Apparatus.—The apparatus, known as the ballistic pendulum, consists essentially of a long arc graduated in degrees and supported with its plane vertical. Hung from the center of this arc are two elastic balls so suspended that they swing freely just above the graduated arc. An arrangement is also provided for tying the larger ball with a thread to the extreme right of the arc. A set of riders on the arc will serve to record the motions of the balls.

Manipulation.—The conditions of collision mentioned in the first paragraph of the General Discussion are approximately secured by using bodies of spherical shape supported by bifilar suspensions, so that their centers are at the same radial distance and in the same plane of vibration.

As it is, however, practically impossible to measure directly the velocities immediately before and after impact, use is made of the fact that if a sphere under the influence of gravity swings through an arc having an amplitude AB , it will have acquired at the lowest point B a velocity the same as if it had fallen vertically through the sagitta h of the arc, and its velocity will be equal to $\sqrt{2gh}$, or it will be proportional to $\sqrt{h}$.

The piece of apparatus to be used in this exercise is, therefore, so constructed that the *ratio* between all three of the heights, viz., the height of *fall* of M before impact and the heights of *rise* of both M and m after impact, may be obtained by means of a graduated arc.

To put the apparatus in adjustment, make the upright truly vertical and then adjust the lengths of the supporting strings until when the balls hang freely *their centers are in the same arc*, and the wire loop carried upon the bottom of the smaller sphere m clears the index No. 1 (with the short finger), but catches the index No. 2 (with the long finger), while the loop on the larger sphere M catches index No. 1. Then complete the adjustments so that when the thread which holds M back is burned, m is driven straight up the arc by the impact with M . When M is tied ready to be released, place index No. 1 in contact with the loop on M and take the reading. Call this position a . Always read the positions of the indexes to $1/100$ of a degree. Next slide index No. 1 down nearly to the point to which M will move after the impact (this point should be approximately located by preliminary trials and designated as reading b) and place index No. 2 a little below the point C to which m will be driven. Then burn the thread, catch M with the hand as it swings back after the *impact* and take the readings b and c . Repeat this procedure recording all readings until the apparatus is in the best possible adjustment, then take six further sets of readings and use the average of these latter in the computations. The ball M should, of course, be started each time from *exactly* the same point on the arc. Unless otherwise directed, let this be the 13° mark. Make two independent determinations of the *zero reading* f of each ball, i. e., the reading when it hangs *alone* (using the proper index). Finally take the reading of index No. 1 when M is at the point of impact d . *This is not the reading when both balls hang together*, but the reading when m hangs freely and M is brought

down so as just to touch m . Make this adjustment four times.

If α represents the difference between the zero reading of M using index No. 1 and the reading at the point of impact d , θ the difference between the zero reading of M and the reading at a , β the difference between the zero reading of M and the reading at b , and ω the difference between the zero reading of m

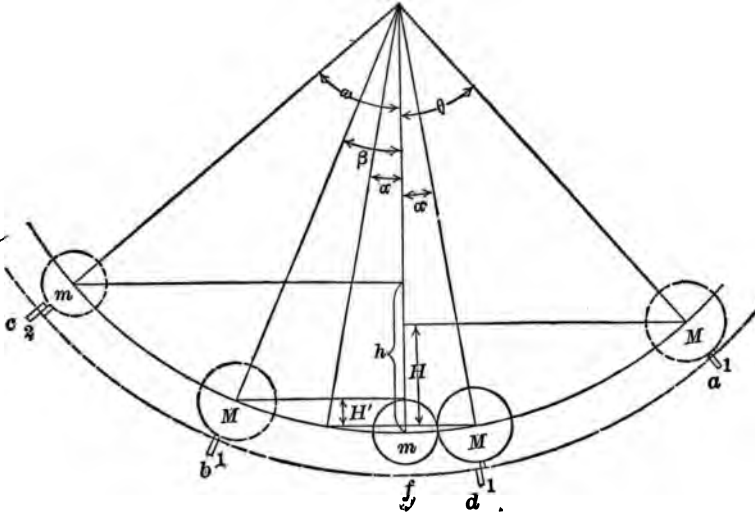


FIG. 8.

by index No. 2 and the reading at c , it is evident that the height H through which M falls before impact, the height h through which m is raised by the impact, and the height H' through which M rises after impact by virtue of the velocity which it retains, are given by the relations,

$$H = r(\cos \alpha - \cos \theta)^*$$

$$h = r(1 - \cos \omega)$$

$$H' = r(\cos \alpha - \cos \beta)$$

Computations.—(1) In the first part of this exercise it is required to verify equation (1) of the General Discussion, viz.,

$$MU = mv + MV \quad (1)$$

* *Note.*—The actual value of r need not be obtained as it cancels out in all computations.

Determine M and m to one tenth of a gram by a balance. Bearing in mind that the velocities U , v and V are proportional respectively to the square roots of the heights of fall or rise H , h and H' , demonstrate the truth of formula (1) with the data obtained in the experiment. Explain the origin of any difference observed between the momentum before and after impact.

(2) Since the ball m is at rest at the start in this experiment, the expression for coefficient of restitution becomes

$$e = \frac{v - V}{U}$$

Compute this coefficient for the balls used.

(3) Calculate the relations of the kinetic energies before and after impact and determine the *per cent. of loss*. In this particular case the *energy* before impact is proportional to MH and after impact proportional to $mh + MH'$.

EXERCISE 14

THE MERCURIAL BAROMETER

Object.—To become acquainted with the construction and manipulation of mercury barometers and with the method of reduction of observed readings to standard conditions.

General Discussion.—By the *barometer height* is meant the height of a column of pure mercury at 0° C. or 32° F., which just balances the pressure exerted by the atmosphere at the *time and place of the observation*. The barometer height is not a true measure of the pressure since the weight of a given column of mercury changes with variations in the value of g , i. e., with altitude and latitude. In order, therefore, to compare barometric pressures at different places by the barometer heights, it becomes necessary to reduce such readings to standard gravity. Standard gravity is taken at latitude 45° and sea-level, where $g = 980.6 \frac{\text{cm.}}{\text{sec.}^2}$.

In the use of a barometer it is always necessary to apply one or more of the following corrections to the observed height, according to the precision that is desired.

1. *Correction for Temperature of Mercury and Scale.*—Let h be the observed height of the mercury at t° . This reading is in error since the mercury is not at the standard temperature, i. e., 0° C. or 32° F. Likewise it is usually in error because the scale is not standard at the temperature of the measurement. (Metric scales are standard at 0° C., English scales at 62° F.) The corrections to be applied to the observed reading h to obtain the *barometer height* may be found from the Smithsonian Physical Tables, or these corrections may be computed from a knowledge of the temperature and the coefficient of expansion of the mercury and the scale.

2. *Correction for Capillarity.*—Because of the capillary depression of mercury in glass the readings of cistern barometers in which the cross-section of the barometer tube is small will be appreciably too low. The magnitude of this error is *inversely* proportional to the *diameter* of the tube, amounting to only 0.025 mm. for tubes 20 mm. in diameter. The correction depends very markedly upon the condition of the glass surface and the

EXERCISE 15

BOYLE'S LAW

Object.—To investigate the relation between the pressure and the volume of a fixed mass of air, maintained at a constant temperature, for pressures greater and less than one atmosphere, i. e., to verify Boyle's law for air.

General Discussion.—Boyle's law asserts that, the temperature remaining constant, the pressure (force per unit area) which a given amount of gas exerts upon the walls of the containing vessel is inversely proportional to the volume which it occupies; symbolically

$$\frac{p_1}{p_2} = \frac{v_2}{v_1}$$

or,

$$p_1 v_1 = p_2 v_2 = p v = c \text{ (a constant).}$$

For a discussion of the deviations from this law and of the kinetic theory of gases the student is referred to the textbook, Duff, fourth edition, pages 151–153, 155–157, or to Watson's Text Book of Physics (1911 edition), pages 144–148, 161–164, where a fuller discussion may be found.

Manipulation.—This exercise consists in measuring the volume of a confined portion of air at *constant temperature* under various pressures. The air is confined over mercury in a graduated tube holding about 25 c.c. and provided with a stopcock at its upper end. The volume of the air in this tube is changed by raising or lowering an open tube connected with it by a piece of flexible rubber tubing.

To facilitate reading the positions of the mercury columns, a mirror with a horizontal scratch to serve as the index line is placed behind both tubes and is movable up and down a graduated bar. Readings on either mercury column are taken by placing the eye and the sliding mirror so that the *top* of the column and its reflection appear to be at the same height as the scratch on the mirror, and then reading the position of the mirror on the graduated bar, using the vernier. The pressure of the

confined gas expressed in centimeters of mercury at room temperature is obtained with sufficient precision by *properly combining the difference in height of the two mercury columns with the uncorrected reading of the barometer.*

The volume of air is obtained from the graduation on the tube by taking the reading on a level tangent to the top of the meniscus and adding the constant correction 0.05 c.c. to allow for the air between this point and the bottom of the meniscus.

Some sources of error in the apparatus are:

1. Leakage of air into or out of the apparatus either by air bubbling up through the mercury or at the stopcock. This error may be detected at any time by bringing the two tubes to the same level and noting whether the volume of the gas has changed.

2. Change in temperature of the apparatus. To obtain the desired precision in the experiment the temperature of the apparatus should not change by as much as a degree. Be careful that no heat is communicated to the enclosed air by the *hands in touching the tube*, or from any other source. Again, any change in volume is accompanied by a temperature change, and therefore it is advisable after any considerable change in volume to wait a minute or two before reading the volume of the air and the heights of the mercury columns.

Procedure.—(1) First record the room temperature, also the air pressure, using the metric barometer.

- (2) Open the stopcock of the "closed tube" and adjust the mercury columns so that a certain amount of air (about 13 cubic centimeters) is contained in the closed tube. Then close the stopcock, being careful to make it perfectly airtight, and read the volume of the enclosed air.

- (3) Next reduce the volume of the gas approximately 0.5 c.c. by changing the height of one or both of the mercury columns, and obtain readings for the volume and the heights of the mercury columns. Repeat this process until pressure and volume have been determined for eight volumes, differing consecutively by about 0.5 c.c.

- (4) Now return to the original volume, recording the heights of the mercury columns to make sure that the mass of air has not changed. Then, increasing the volume by steps of 0.5 c.c. each, obtain eight sets of pressure and volume readings. Next return to the original volume and record pressure as before.

Computations and Plots.—(1) Plot a curve showing the relation between the various pressures and the corresponding volumes of the gas, using as ordinates the values of the absolute pressure (p) in millimeters of mercury and as abscissas the volume (v) in c.c. (2) Plot also a curve between p and $1/v$.

Note.—See sample curve sheet posted on the Bulletin Board.

Problems.—(1) From the coordinates of a point on the plot for p and $\frac{1}{v}$ compute the values of the constant $c = pv$ when the volume is expressed in cubic centimeters and the pressure is expressed (1) in centimeters of mercury, (2) in grams per square centimeter, (3) in dynes per square centimeter. Density of mercury may be taken as 13.55 gram/cm.³ at room temperature and $g = 980.2$ cm./sec.².

(2) How does the value of pv depend on the temperature of the gas?

(3) With a given mass of gas, what percentage change would be produced in the value of pv if the temperature changed from 20° to 21° C.?

EXERCISE 16

PHOTOMETRY

Object.—Introduction to the fundamental principles of photometry by the measurement of the candle-power and the efficiency at several voltages of three filament lamps using a photometer bar provided with a Bunsen and with a Lummer-Brodhun screen.

General Discussion.—For a few preliminary remarks on the subject of photometry the student is referred to Duff, fourth edition, §§ 639–640 and §§ 723–724. Additional references: Barrow's *Light, Photometry and Illumination*; Circular of the Bureau of Standards No. 15 on the International Unit of Light.

The law of inverse squares as explained in the above-mentioned paragraphs depends on the assumption that the light diverges from a point source. This is not strictly true with an incandescent lamp, but is more closely approximated the greater the distance from the lamp. For this reason the screen should never be closer than 50 cm. to either lamp. (The farther away the better.)

The efficiency of any device for the transformation of energy is usually given as the ratio of output to input, both expressed in the same units, and may, therefore, be expressed as a fraction or a percentage. In the case of luminous efficiency we have no commonly recognized mechanical equivalent of luminous energy. This efficiency may be expressed as candle power per watt, but is generally given as the watts per candle power, which is not, in the strict sense of the word, an efficiency at all.

Apparatus.—A divided bar 250 cm. in length supplied with two lamp holders, a Bunsen screen and a Lummer-Brodhun contrast screen; a standard 16 c.p. carbon lamp; three unknown filament lamps; two variable resistances; a voltmeter and an ammeter; and a storage battery supply of 140 volts.

Manipulation.—Place the standard lamp in the socket at the right end of the graduated photometer bar with the fiducial line

of the lamp facing the photometer screen, and adjust the series resistance until the voltmeter across the terminals indicates the voltage stated on the standard lamp. The voltmeter is connected to a double-pole double-throw switch (D.P.D.T.) so it can be used to measure voltage drop across the standard lamp or the lamp under test. The lamp to be tested should be screwed into the socket at the left end of the bar and the resistance adjusted for any desired voltage. An ammeter in series with the lamp under test indicates the current flowing through it. See additional instructions to be found with the photometer.

The candle power of the unknown lamp using the Bunsen screen should be determined at any voltage by adjusting the position of the photometer box between the two lamps until the star of the screen disappears or becomes equally faint on both sides of the screen, as seen in the mirrors at the back of the screen. The position of the screen is then read on the photometer bar. Two independent settings should be made in this way, then the screen should be reversed and two more independent settings taken. The candle power of the unknown lamp is to be computed from the average of each set of four measurements. Record carefully at the time of each setting the terminal voltage of the lamp under test and the current flowing. Leave space on the data sheet for recording in columns the computed value for the watts, the difference between 250 and the average bar reading, the multiplying factor, the candle power and the watts per candle power. The same procedure should be followed with the Lummer-Brodhun contrast screen with the exception that the screen should be set for equal contrast instead of equal brightness. See Barrows, page 89.

Data should be obtained in this way for the unknown 117 volt carbon and 120 volt tungsten lamps, varying the voltage from 90 to 120 volts in steps of five volts each. In obtaining data for the 52 volt carbon lamp, shunt the ammeter, reduce the terminal voltage to 42 and increase in steps of four volts to 62 volts. Settings on the carbon lamps should be made with the Bunsen screen, the Lummer-Brodhun being used with the tungsten lamps.

Computation and Plots.—Compute by a slide rule the candle power and efficiency (watts per candle power) of each lamp at each of the voltages used. Plot on the same sheet of cross-

section paper the curves obtained for these lamps, using efficiency as ordinates and voltage drop across the terminals of each of the three lamps as abscissas. Select the scales in such a way as to obtain plots of reasonable size without unduly magnifying any errors.

Note.—See sample sheets posted.

EXERCISE 17

THE SPECTROMETER.—INDEX OF REFRACTION

Object.—(1) To obtain practice in the adjustment of a spectrometer. (2) To measure one angle of a prism. (3) To determine the index of refraction of a glass prism for light of two wavelengths, i. e., yellow of sodium and red of lithium.

General Discussion.—One of the most valuable instruments of a Physical Laboratory is the Optical Circle, or Babinet's Goniometer. This name is applied to it when it is used to measure the angles of crystals, but when it is constructed of large size with divided circle and verniers for the measurement of prism angles and the deviation of light rays, it becomes a spectrometer. When used to investigate and compare spectra, it is often called a spectroscope. In all of these operations, however, it serves essentially to measure the angle between the directions of two beams of light.

A spectrometer consists primarily of an observing telescope and a collimator tube attached to a base in such a way that the telescope may be rotated about a vertical axis bearing a horizontal graduated circle while the collimator, lying in the same plane as the telescope, is stationary and, like the telescope, may be adjusted to point toward the axis of the graduated circle. The telescope is supplied with cross-hairs at its principal focus and two verniers by means of which its position on the graduated circle may be read. Two verniers are provided in order to correct for the angle of eccentricity. The telescope may be held in any desired position by a clamp and moved slowly by a tangent screw. The collimator is provided with an adjustable metal slit with straight parallel edges placed in the principal focus of the collimator objective.

Before proceeding with the exercise an examination of the construction of the instrument to be used should be made, special care being taken to become familiar with the object of the various screws which serve either to clamp some part of the instrument or to give that part a slow motion. The movable parts of the instrument will, in general, be provided with a

clamping arrangement and a fine adjustment screw. Before moving any part of the apparatus, therefore, care must be taken to release the clamping screw. All parts should move easily and *should under no circumstances be forced*. In clamping, the screw should not be forced, but turned with care. If force is used the instrument may be damaged.

Before making any measurements the divided circle and the verniers should be examined, and the value of the smallest subdivision of the principal and vernier scales determined. When entering an observation in the notebook, the positions of the two verniers should always be recorded.

To Focus the Observing Telescope.—Direct the telescope toward a bright object and move the eyepiece in or out by a twisting motion until the cross-hairs are distinctly in focus. Now focus the telescope as a whole on an object at a distance of 100 feet or more so as to ensure approximate parallelism of rays [without altering the previous adjustment of the eyepiece] until the image of the object lies in the plane of the cross-hairs, i. e., until there is no parallax.

To Focus the Collimator.—Direct the observing telescope toward the collimator so that their axes are parallel and intersect the vertical axis through the center of the circle. Illuminate the slit and regulate its size until it is fine but still distinctly visible through the observing telescope. Focus the *collimator* by moving the slit towards or away from the *collimating* lens until the image of the slit is sharp and there is no parallax between its image and the cross-wires, *but do not in any way alter the original adjustment of the observing telescope*. The slit will then be at the principal focus of the collimating lens, and the beam of light between the collimator and the telescope will be parallel. If the slit is not vertical it should be turned until it is so.

There still remain a number of fine adjustments which should be made for work of a high precision, but for the purposes of this exercise the two described will be sufficient.

Reference: Watson, A Text-Book of Practical Physics.

Index of Refraction.—Any monochromatic light is propagated with different velocities through different media, the ratio of its velocity in a vacuum to its velocity in any other medium being called the *absolute index of refraction* of this medium. It is not, however, practicable to compare these velocities directly, but

use is made of the fact that when light passes from one medium into a second, its direction of propagation in the second medium as compared with its direction in the first is a function of the angle of incidence upon the bounding surface and of the indices of refraction of the two media. If i is the angle of incidence, r the angle of refraction, μ_1 and μ_2 the absolute indices of refraction of the two media, then

$$\sin r = \frac{\mu_1}{\mu_2} \sin i$$

In laboratory measurements the first medium is usually air. If air is taken as the standard medium instead of a vacuum, then $\mu_1 = 1$, and the *relative index of refraction* of the second medium referred to air as standard becomes

$$\mu = \frac{\sin i}{\sin r} \quad (1)$$

When the light is transmitted through a medium in the form of a prism at the angle of *minimum deviation*,

$$\mu = \frac{\sin \left(\frac{\Delta + \alpha}{2} \right)}{\sin \frac{\alpha}{2}} \quad (2)$$

where Δ stands for the angle of minimum deviation, and α is the angle of the prism.

Apparatus.—Spectrometer, prism, sodium-burner and lithium solution.

Manipulation.—(1) *Adjust the Spectrometer as Described in the General Discussion.*

(2) *Measurement of Prism Angle.*—The work consists in finding, as accurately as possible with the instrument, a particular angle of the given glass prism by the following method:

Stand the prism on the center table or platform with its edges vertical and with the faces whose angle is to be determined about equally inclined to the axis of the collimator C . Before recording the directions of the rays reflected from either face it will be well to turn the telescope so as to receive the rays reflected from one face and then from the other and observe whether it will be

possible in each position to read the two verniers. Sometimes one vernier may come under the collimator or telescope and be inaccessible. This difficulty may be obviated by altering the position of the prism on the table. Place the Bunsen burner supplied with the sodium tip at L (Fig. 9) *not nearer than three*

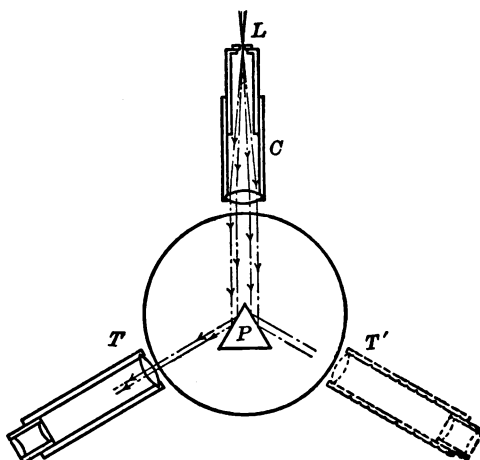


FIG. 9.

inches from the slit. Turn the prism P toward the collimator C so that the beam of light will fall on the two faces of the prism which form the angle to be measured. Clamp all the movable parts of the spectrometer *except* the telescope. Set the telescope in the position T so that the intersection of the cross-wires is near the image of the slit, then clamp it and adjust carefully by means of the tangent screw until the intersection of the cross-wires coincides with the center of the image of the slit. Read the position of the *two* verniers attached to the telescope. In this way make four independent settings. Turn the telescope to the position T' and set it on the image of the slit reflected from the second face, and take four readings as before. The average angle through which the telescope has been turned is twice the prism angle.

(3) *Angle of Minimum Deviation, Index of Refraction.*—Having determined the angle of the prism, proceed to find first the minimum deviation for sodium light. Place a Bunsen burner supplied with a sodium burning tip, arranged to give a sodium flame, about three and one half inches beyond the slit. Without alter-

ing the adjustments of the spectrometer, arrange the collimator and prism so that light coming from the former will be refracted through the latter. Find the refracted image of the slit in the first place by the unaided eye, and follow the image while the platform or prism is slowly turned around until the position of minimum deviation is found approximately. Bring the telescope into this position *T* of Fig. 10 and locate the image of the slit in

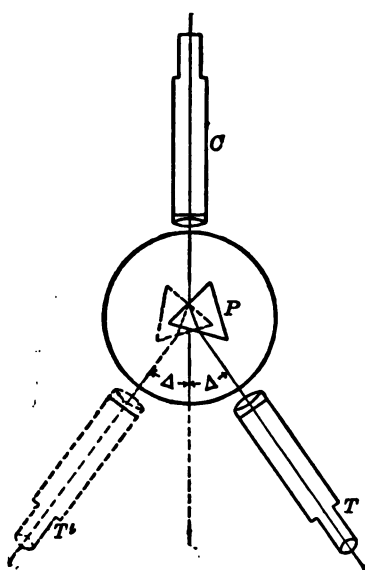


FIG. 10.

the telescope. Watch this image while the prism is slowly turned. If the direction of rotation is properly chosen, the image will move slowly in the direction of least deviation, come to a standstill, and move back again. Leave the prism in the position for which the deviation is least, place the cross-wires of the telescope approximately on the image, clamp the telescope and set the cross-wires more accurately by means of the slow motion screw. Now turn the prism backwards and forwards, resetting the telescope until the center of the image slit comes just up to the center of the cross-wires, but does not pass beyond it. Make

four independent settings, each time reading both verniers. Turn the prism so that it produces minimum deviation on the opposite side of the line of collimation, as indicated by dotted lines in Fig. 10, and again record the angle, making four independent settings as before. One half the average angle through which the telescope is turned gives the angle of minimum deviation of this prism for sodium light.

Repeat these operations, using the red light of lithium obtained by burning lithium chloride on a platinum wire in a Bunsen flame.

After computing the angles of minimum deviation for the average wave-length of sodium light (5893×10^{-8} cm.) and for the red of lithium light (6078×10^{-8} cm.), also the angle of the prism, compute the index of refraction of the given specimen of glass for light of these two wave-lengths.

Arrange the data in neat and convenient tables. Be especially careful to record the readings of verniers *A* and *B* in *separately headed* columns to avoid mistakes in the interpretation of these readings.

Note.—The student will be held responsible for the proof of formula (2) as well as for the proof of the method employed in determining the angle of the prism, i. e.,

$$TPT' = 2\alpha \quad (\text{Fig. 9}).$$

These proofs should be incorporated in the report on this exercise.

EXERCISE 18

THE SPECTROMETER.—DISPERSION CURVE

(Read Exercise 17 before performing this experiment)

Object.—The object of this exercise is to determine the dispersion curve of a prism by the use of a precision spectrometer, and to determine the wave-length of certain unknown lines by means of this curve.

General Discussion.—For a general description of the instrument itself the student is referred to Exercise 17. The dispersion or prism curve is a graphical representation of the relation that exists between the angle of deviation and the wave-length, the prism being in the position for minimum deviation for the sodium lines $\lambda = 5896$ and $\lambda = 5890$ Ångström units.

Note.—Wave-lengths are now usually expressed in Ångström units, also called tenth-meters. 1 tenth-meter = 10^{-7} meter = 0.000,000 1 millimeter.

In the case of *glass* prisms it is essential to know the dispersion curve of any particular prism in order to obtain the wave-lengths of unknown lines, as a slight difference in the composition of the glass of two prisms may produce a decided alteration in the dispersion curve, even though the angles of the prisms are exactly the same. For glass prisms these curves are smooth and even, whereas in the case of substances with selective absorption, i. e., substances whose molecules have natural periods which are the same as those of certain of the light-waves, the curve may be quite irregular.

Note.—In the case of a diffraction grating no curve is necessary, as gratings always give normal spectra, i. e., spectra in which the deviation is proportional to the wave-length. This enables us with such a grating to compute the wave-length of any line when its angle of deviation, the order of the spectrum, and the distance between the rulings of the grating are known. The formula used in this case is

$$\lambda = l \sin \delta_1 = l/2 \sin \delta_2 = l/3 \sin \delta_3, \text{ etc.,}$$

where δ is angle of deviation, l the distance between the rulings, and the subscript refers to the order of the spectrum. The general formula for the spectrum of the n th order is $\lambda = l/n \sin \delta_n$.

References: Watson, A Text-Book of Practical Physics. Nichols and Blaker, A Laboratory Manual of Physics and Applied Electricity.

Apparatus.—The apparatus consists of a large Geneva Society spectrometer with all the necessary accessories for producing flame and vacuum tube spectra.

Manipulation.—In order that the spectrometer may be used for precise measurements, the following adjustments must be carefully made:*

1. The telescope must be adjusted for parallel rays by first adjusting the ocular on the cross-wires and then focusing upon a distant object until rid of parallax.

2. The collimator must be adjusted for parallel rays by sighting the telescope previously adjusted for parallel rays upon the illuminated slit and pushing the slit in and out until no parallax exists between its image and the cross-wires.

3. The axes of the telescope and collimator must lie in a plane perpendicular to the axis of the instrument.

4. The axes of telescope and collimator must intersect the prolonged vertical axis of the instrument.

5. The prism must be set by means of the base screws so that its faces are perpendicular to the plane in which the axes of the telescope and collimator lie.

After having completed these adjustments remove prism and point the telescope directly towards the illuminated slit. Carefully read both verniers, making two independent settings. The readings should be recorded in two columns headed Vernier A and Vernier B, A being the one farthest away from the telescope. In a third column the corrected reading should be entered to

* *Note.*—Adjustments 2, 3 and 4 have already been made and should not be disturbed. Adjustment 1 may, under these conditions, be made by sighting directly upon the illuminated slit which now takes the place of the infinitely distant object. In order to make Adjustment 5 the telescope should be clamped, making an angle of about 90° with the collimator. The slit should be illuminated and that face of the prism used as a reflecting surface which is perpendicular to the line connecting two of the levelling screws of the prism stand. Adjust these screws until the slit appears centrally in the field of view. Now turn the prism table and by means of the remaining screw adjust the second face of the prism until the slit again appears centrally in the field of view. Check the adjustment by turning the prism back to its original position and make any slight readjustment which may still be necessary.

°, ', ", the degrees corresponding to the reading of Vernier *A* and the minutes and seconds being the average of *A* and *B*. Similar readings and records should be made for all subsequent positions of the telescope. Now place the prism in the position of minimum deviation for the sodium lines, so that the rays are refracted towards the right, being sure that the Bunsen burner is at least three inches from the slit. Keep the prism in this standard position. Connect in succession the hydrogen and helium spectra-tubes to the secondary of the induction coil by fine wires. In each case adjust the tube stand so that the light will fall directly upon the slit. The tubes should be run only for as short a period as may be necessary to obtain the position of the lines. Use the mercury lamp for the mercury spectrum. Record the position of all the bright and distinct lines of hydrogen, helium, and mercury.

TABLE OF WAVE-LENGTH

Element	Nature and Color of Line	Wave-length
Sodium.....	Yellow (br)*	{ 5896.2 5890.2
Lithium.....	Red (br)	6708.2
Thallium.....	Green (br)	5350.6
Rubidium.....	Red (m)	7806.0
	Violet (m)	{ 4215.7 4202.0
Caesium.....	Red (m)	6723.6
	Orange (f)	6250.
	Yellow (f)	6010.6
	Blue (m)	{ 4593.3 4555.4
Hydrogen.....	(C) Red (br)	6563.0
	(F) Green (br)	4861.5
	(G) Violet (br)	4340.7
Helium.....	Red (m)	7065.5
	Red (m)	6678.4
	Yellow (double) (br)	5876.0
	Green (f)	5047.8
	Green (br)	5015.7
	Greenish-blue (m)	4922.1
	Blue (m)	4713.3
	Blue (br)	4471.6
	Violet (double) (br)	4388.5

* (br) = bright, (m) = medium, (f) = faint.

Computations and Curve.—From the values on the third data column compute the deviation of all the measured lines. Plot these values to hundredths of a degree as abscissas and the known wave-lengths, given in the appended table, as ordinates (see sample curve posted). Draw curve very carefully with a hard, sharp lead pencil and determine by means of it the wave-lengths of the principal Hg lines.

EXERCISE 19

THE CHEMICAL SPECTROSCOPE

(Read Exercise 17 before performing this experiment)

Object.—The object of this exercise is to familiarize the student (1) with the adjustment and use of the chemical spectroscope; (2) with the general character of spectra; (3) with the elementary principles of spectrum analysis.

General Discussion.—Duff, pages 627–629, also 631–644 (fourth edition), also Edser's *Light*. For additional references to spectrum analysis see: Landauer's *Spectrum Analysis*; Baly on *Spectroscopy*; Watt's *An Introduction to the Study of Spectrum Analysis*; Wood's *Physical Optics*; Kayser's *Handbuch der Spectroscopie*. The construction of the Kirchhoff-Bunsen spectroscope is very similar to that of the spectrometer, the only necessary addition being a scale tube containing a finely divided scale at the principal focus of its lens. This scale tube is so placed that the light from the scale is reflected by one face of the prism into the observing telescope, where an image of the scale is formed above the spectrum under investigation. The Hoffman type of instrument differs from the above type by making use of a direct-vision prism, thereby enabling the collimator and telescope to lie in the same straight line. In both instruments the telescope is movable so as to enable the observer to view the entire spectrum. The collimator and scale tube are rigidly fastened to the table of the instrument, the only adjustments being for focusing the slit and scale respectively. *The adjustment of the collimator for parallel rays has been made and should not be changed.*

KINDS OF SPECTRA.—Emission spectra are of three types, continuous, banded, and line spectra.

Continuous Spectra.—These are characteristic of incandescent solids and liquids, the extent and intensity of the spectrum depending upon the temperature to which the substance is heated.

Banded Spectra.—These are particularly characteristic of compounds. They often present striking peculiarities. With small

dispersion they are, in certain cases, sharply and intensely defined on one edge and fade away towards the other, as in the case of the barium compounds. In others, as in the strontium compounds, they have a maximum intensity at the center and fade away on both sides. When observed with greater dispersion, each band is seen to be made up of a large number of very fine lines which come closer together as the sharp edge is approached. The spectra of the halogen salts of mercury and of the bluish green cone of a Bunsen flame offer excellent examples of banded spectra.

According to the kinetic theory, banded spectra are, generally speaking, determined by the structure of the molecule of the compounds as a whole. For this reason banded spectra can be observed only so long as the compound remains undissociated. Line spectra (see below) are determined by the nature of the atom. In both cases the radiation has its origin in the vibration of the electrons. When the temperature is sufficiently high to dissociate the compound into its elements, a line spectrum of one or more of the constituent elements is obtained. The temperature of the Bunsen flame is sufficient to dissociate many compounds, particularly the nitrates, carbonates and chlorides of the alkalis and alkaline earth metals, in which cases the bright lines of the metallic elements alone appear.

Line Spectra.—These consist of sharp, well-defined bright lines separated by dark intervals, and are characteristic only of elements in the state of luminous gas or vapor.

Methods of Producing Characteristic Line Spectra.—Spectra may be produced by introducing substances, which are under normal conditions in a solid or liquid state, into the non-luminous Bunsen flame, or, if more heat is required to dissociate and vaporize them, into the electric arc, the spectra so produced usually being characteristic of the metallic elements present. In the case of uncombined metals it is often convenient to pass an electric spark between terminals of the metal to produce its spectrum. Gases are usually studied by passing an electric discharge through a specially constructed tube containing the pure gas under a very low pressure. According to the mode of excitation, spectra are spoken of as flame, arc, spark and vacuum tube spectra.

Spectrum Analysis.—The brilliant researches of Bunsen and

Kirchhoff during the years 1859–1862 laid the foundation for the marvelous developments that have taken place in spectrum analysis and in astro-physics. They pointed out the sensitiveness of the spectroscope as a means of detecting the presence of an element, and in fact Bunsen in 1860 discovered rubidium and caesium by this means. Later thallium, indium, gallium and helium were discovered with the spectroscope, and it has been used to identify as elements, argon, krypton, neon and radium.

To detect the presence of an element in a compound, it is necessary to dissociate the compound and render the element that is to be detected luminous as a vapor. The characteristic lines of the element thus obtained are found to be independent of the compound of the element used, though the number of lines that may be distinguished depends to some extent upon the temperature and the mode of excitation. For example, sodium which in the Bunsen flame gives the usual lines in the yellow, gives at the high temperature of the oxy-hydrogen flame certain additional lines in the green and blue. As the temperature of the Bunsen flame is sufficient to dissociate many salts into their elements, the flame spectra of these compounds will give information chiefly regarding the metallic elements present.

Apparatus.—The apparatus consists either of a Kirchhoff-Bunsen or a Hoffman direct-vision spectroscope and all the necessary accessories for producing flame and vacuum tube spectra.

Manipulation.—In chemical spectroscopes the prisms are usually permanently adjusted by the makers in their position of minimum deviation for sodium light, for which reason *the position of the collimator with respect to the prism should not be altered.*

Illuminate the slit with sodium light placing the burner at least $2\frac{1}{2}$ to 3 inches away from it, and, if necessary, adjust the telescope until the image of the slit is perfectly sharp. Having done this, illuminate the scale by means of the small incandescent light and focus the scale tube until all parallax between the graduations of the scale and the sodium line has been removed. See that the scale is horizontal and perpendicular to the image of the slit. It should be so placed that the image* of the *D* (sodium)

* *Note.*—In case the slit is so constructed that only one jaw is movable, then the image of the immovable side of the slit should be placed at the reference mark, and the position of the corresponding side of all the lines should be recorded. If, however, both jaws partake in the motion when the slit is opened or closed, then the centers of all the lines must be recorded.

line coincides with the graduation marked 3.5 in the Schmidt and Haensch instrument and 5 in the Hoffman. If this is not the case, notify the instructor. After these adjustments have been made, proceed as follows:

1. Remove the sodium tip and examine the flame spectra given by lithium, thallium, rubidium, and caesium, being careful *not* to *interchange* the *platinum wires*, each of which should be used only in accordance with the label upon it. Record in tabular form the element, color and position of all the stronger, well-defined lines. In order to examine the entire spectrum, the telescope will have to be moved by means of the screw adjustment. The sodium line will probably be present in every flame spectrum, but sodium should only be considered as a constituent of the solution if its line appears very pronounced.

2. Having examined the solutions, adjust the Bunsen burner so as to give a bright greenish-blue cone.

Note the banded spectrum emitted, which is due to hydrocarbons and is known as the "Swan spectrum." Record the positions of the edges of each of the five bands, noting in each case which edge is the more intense.

3. In succession connect the hydrogen and helium tubes by means of fine wires to the secondary of the induction coil. Place them on the stand in such a position as to be in line with the axis of the collimator and actuate by closing switch. The current should be turned on only as long as is absolutely necessary to record the positions of the principal lines, as excessive heating may injure the tube. Use the mercury vapor lamp for the mercury spectrum.

Computation and Plot.—(1) Map on a sheet of cross-section paper the spectra of the solutions; the Bunsen cone; the hydrogen, helium and mercury tubes. Lay off parallel to the shorter edge of the paper the scale as seen in the eyepiece of the instrument as abscissas, and draw vertical lines about five small spaces in length to represent the positions of the various lines and bands.

- (2) Using the lines of the known elements, whose wave-lengths are given in the table appended to Exercise 18, plot the calibration curve of the instrument. This plot should be placed on the same sheet on which the spectra are recorded and should have the same scale of abscissas. The scale of ordinates chosen should permit of convenient plotting to three significant figures and

should be expressed in Ångström units. This curve, as well as the map of the spectra, is to be worked up neatly in pencil as part of the final report on the exercise.

Note.—Before attempting Parts 1 and 2 the student should consult the sample sheet posted.

(3) From the calibration curve determine the wave-length of the lines and edges of the bands found in the Swan spectrum and the lines in the mercury spectrum. Record these results in the fourth column of the table on the data sheet.

EXERCISE 20

FOCAL LENGTH OF LENSES AND LENS COMBINATIONS

Object.—To determine the focal length of a convex and concave lens by optical methods, also to determine the focal length of a lens combination.

General Discussion.—The *principal focus* of a lens is that point on the principal axis through which all rays, parallel to this axis of incidence, will pass after refraction through the lens. This point is well defined only with a lens of small aperture and when monochromatic light is used. If the lens is of large aperture, the rays incident near the edge will be brought to a focus sooner than those incident near the principal axis, thus giving *spherical aberration*. With white light, owing to dispersion, *chromatic aberration* is produced. Both aberrations may be practically eliminated by a suitable combination of lenses, thus permitting of a definite focus with white light.

The principal planes of a symmetrical optical system are characterized by the property that in these planes object and image are congruent; and, therefore, any straight line drawn parallel to the axis will intersect these planes in a pair of conjugate points. The principal points H , H' are the points where the principal axis crosses the principal planes.

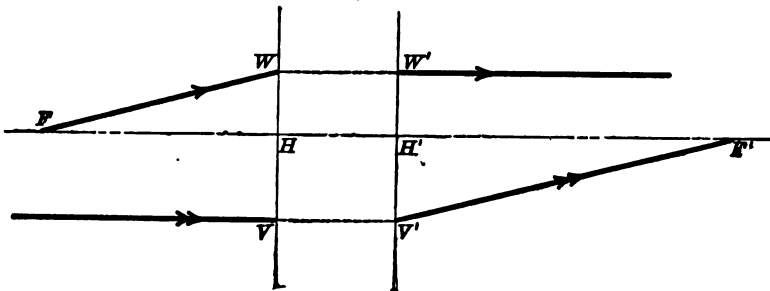


FIG. 11.

To any ray in one region (object-space or image-space) which goes through the focal point belonging to that region, there will

correspond a ray in the other region which is parallel to the principal axis, and the rectilinear portions of the path of the ray in these two regions will intersect in a point lying in the principal plane of that region to which the focal point in question belongs, as is illustrated in Fig. 11 at W and V' . Likewise in Fig. 11 $f = FH =$ primary focal length, $f' = F'H' =$ secondary focal length.

The following treatment presupposes that the lens or lens system is surrounded by a medium of constant refractive index, viz., air. For a *thin lens* the focal length is the distance from the

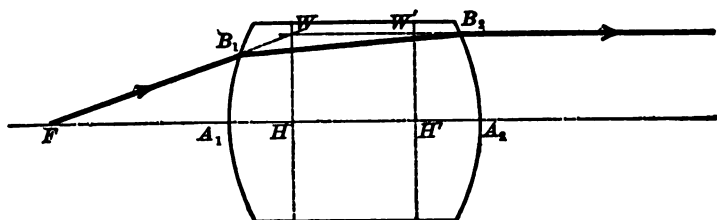


FIG. 12.

principal focus to the surface of the lens, provided its thickness is negligible in comparison with its focal length. For *thick lenses* and *lens combinations* the focal length is measured from the principal points, see Figs. 12 and 13: $FH = f = f' = F'H'$ without regard to the signs.

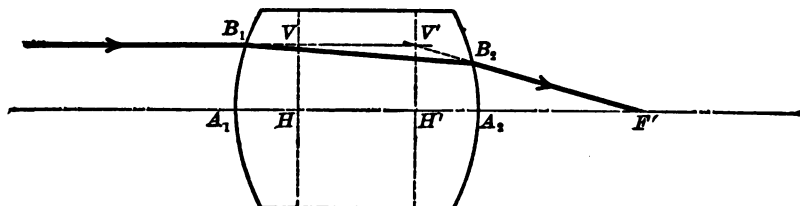


FIG. 13.

When a lens system is surrounded by the same medium on both sides, the principal points (H , H') are coincident with the so-called nodal points. The latter possess the following property: If a ray enters the lens from any direction toward one nodal point, N , it will emerge from the lens parallel to its original course, as if it had come directly from N' , the other nodal point, see Fig. 14.

For thick lenses and lens combinations the values of p and p' in the general lens formula

$$\frac{1}{p} + \frac{1}{p'} = \frac{1}{f}$$

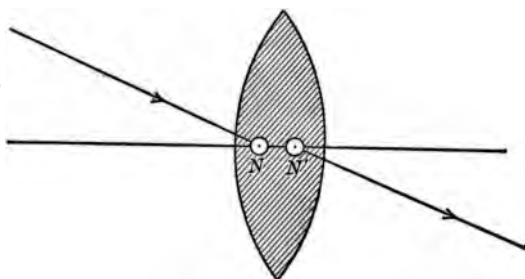


FIG. 14.

are the distances of object and image, respectively, from the principal points, or from the nodal points since the surrounding medium is the same on both sides. See Fig. 15.

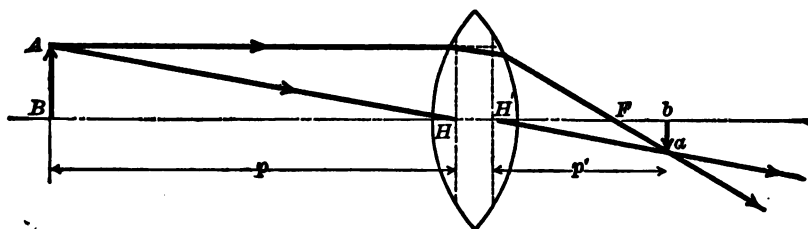


FIG. 15.

In the case of a concave lens, it is evident from an inspection of Fig. 16 that the incident light is made more divergent by the lens. The above formula, therefore, becomes in this case

$$\frac{1}{p} - \frac{1}{p'} = -\frac{1}{f}$$

The significance of the negative sign in this formula is that the principal focus is virtual, its distance from the lens being measured in a direction opposite to that in which the light actually travels.

Reference: Southall, Mirrors, Prisms and Lenses.

Apparatus.—The apparatus consists of an optical bench with a centimeter scale engraved upon it and supplied with the

necessary lens and diaphragm holders; a positive lens; a negative lens; and a lens combination consisting of two lenses separated

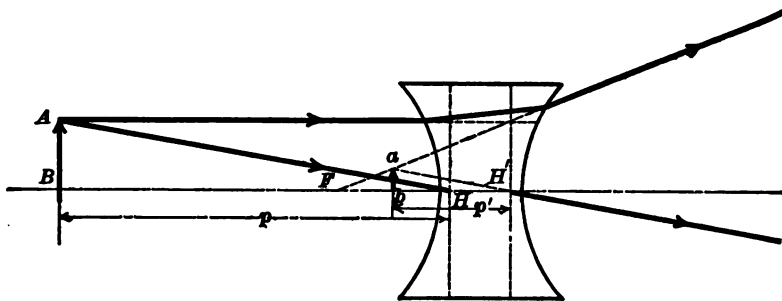


FIG. 16.

by an air space; three ground glass plates, one of which has a black letter upon it; two metal diaphragms; a telescope; a pair of calipers.

Manipulation and Computations.—This exercise presents four methods for the determination of the focal length of *positive* lenses, and one method for the determination of the focal length of a thin *negative* lens. Two of the former are applicable to the case of thick lenses.

A simple method for *thin* positive lenses is to place the carriage on which the lens is mounted on the optical bench between the window and the receiving translucent screen, and to slide the screen along the bar until a sharply defined image of a distant object is formed upon it. (See note below.) Since the object

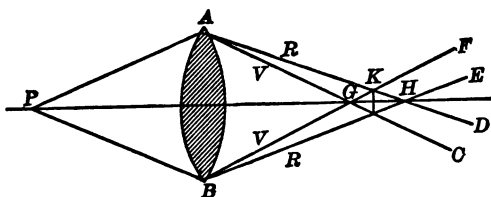


FIG. 17.

Note.—As non-achromatic lenses are used in these experiments, the beginner must be careful in deciding where the screen should be placed for a sharp image. In Fig. 17 *AB* represents a simple non-achromatized lens, *P* an illuminated point, and *PA* and *PB* the limiting rays of white light which strike the lens. These, when refracted through the lens, become dispersed, and two

is far away (practically at an infinite distance), the distance of the screen from the surface of the lens equals the focal length of the thin lens.

METHOD I.—For a *thin positive lens* the focal length may be measured by the use of the Law of Lenses. If an object be placed on one side of the lens and a screen on the other, the screen may be adjusted until the image is sharply defined. It can be shown that under these conditions

$$\frac{1}{p} + \frac{1}{p'} = \frac{1}{f} \text{ or } f = \frac{pp'}{p + p'} \quad (1)$$

where p and p' are the distances from the lens to the object and image respectively.

Place the ground-glass diaphragm with the black letters upon it in the mounting so that it is brightly illuminated by the incandescent lamp. With the diaphragm about 40 centimeters from the lens, move the ground-glass screen until a distinct image is formed. Read the positions of all the indices. Make *four* independent settings of the screen, keeping diaphragm and lens stationary. With the average values of p and p' compute f .

METHOD II.—For *thin positive lenses*: Focus the telescope on a very distant object so that there is no parallax between the image of the object and the cross-hairs of the telescope. Place a ground-glass screen with the black letter on it on the optical bench some distance in front of the telescope. Between the screen and the telescope place the lens, and adjust the distance from the screen until there is no parallax between the cross-hairs and the image of the black letter. Record the position of the screen and lens, making four independent settings, and compute the focal length f .

Show by a carefully drawn diagram how f is obtained by this method.

pencils of colored rays DAC and FBE are obtained. AD and BE will, therefore, be red rays, and AC and BF violet, since the latter are more refracted than the former. It will be readily seen from the figure that any point between G and H may be said to be the image of P . If the screen be placed at G , the image on it will be fringed with a reddish tint, while if placed at H , it will have a violet border. Between these two points, then, there will be a point K where the red and violet rays overlap, and the image will be of one color. The screen should, therefore, be placed at this point.

METHOD III.—The two methods just described for finding the focal length of a lens *cannot* be applied to *thick* lenses or *lens combinations*, since in these cases f , p and p' must be measured from the principal points instead of from the surfaces of the lens. Methods III and IV enable the focal length of a *thick* lens to be measured with precision, although the position of the principal points is unknown.

For a positive lens, if p and p' are the distances of the object and image respectively from the principal points (see Fig. 15) and f is its focal length, then

$$\frac{1}{p} + \frac{1}{p'} = \frac{1}{f}$$

For a real image p and p' are on opposite sides of the lens.

If the object and screen are at a fixed distance $l \geq 4f$ apart, a lens of focal length f may be placed between them so that they are in conjugate positions with regard to it. (See Fig. 15.) Evidently if $l > 4f$, another position of the lens can be found giving a real image of the object. This will be the case when the lens is placed so that the values of p and p' are interchanged. Calling p_1 and p_1' the values which p and p' assume for this new position of the lens, and d the distance through which the lens has been moved, then,

$$p_1 - p = p' - p_1' = d$$

or

$$p' - p = p_1 - p_1' = d \quad (2)$$

also

$$p + p' + HH' = l \quad (3)$$

Now if μ is the index of refraction of the glass forming the lens and t represents its thickness, an approximate value for the distance between the principal points H and H' can be shown to be

$$HH' = \frac{\mu - 1}{\mu} t \quad (4)$$

Transforming equation (1) by means of equation (2) and substituting for $(p + p')$ the value obtained by equations (3) and (4), it follows that

$$f = \frac{pp'}{p + p'} = \frac{\frac{(p + p')^2 - d^2}{4}}{p + p'} = \frac{\left(l - \frac{\mu - 1}{\mu} t\right)^2 - d^2}{4 \left(l - \frac{\mu - 1}{\mu} t\right)}$$

or

$$f = \left[\frac{l^2 - d^2}{4l} \right] - \left[\frac{\mu - 1}{\mu} \times \frac{l^2 + d^2}{4l^2} \right] t. \quad (5)$$

Set the illuminated object diaphragm and the screen a distance of 120 to 150 cms. apart. Place the single thin lens previously used between them and obtain a mean value for d from four independent settings of the *lens* in each position. This distance d is, of course, the difference between the average index readings of the lens in the two positions. Obtain value of t by means of the caliper, and then, assuming $\mu = 1.52$, calculate the focal length of the lens by equation (5).

Compare the result obtained with those of Methods I and II.

Why can this method not be used for a lens combination?

METHOD IV.—*Applicable to thick positive lenses and lens combinations.* The principle of the method is as follows: An image of a very distant object is formed on a screen at the principal focus of the lens. If another object is then brought near the lens on the same side as the distant object to such a position that a real image is formed on the screen of the *same size* as the object, the *screen* must have been moved through a distance equal to the focal length of the lens or lens combination.

Operation 1.—Focus with the lens combination some distant object sharply upon the *plain ground-glass screen*. Make four independent settings of the *screen* and record the positions of its index each time. Record also the position of the lens index.

Operation 2.—Keeping the lens combination in exactly the same position as in Operation 1, substitute for the distant object the circular metal diaphragm having a circular aperture. This diaphragm, mounted in a lens holder with a plain ground-glass screen, should be illuminated by an incandescent lamp. Place before the receiving screen the metal diaphragm with rectangular hole whose shorter side is equal to the diameter of the circular hole used as object. Now move the *object* and the *screen* to such positions that the object and image are of the same size, i. e., the image just touches the top and bottom of the rectangular opening on the screen. In this way make four independent settings of the screen and object, recording each time the position of the *screen* index.

Prove that the difference between the average positions of the screen index as obtained in Operation 1 and as obtained in Operation 2 is the true focal length of the lens combination.

METHOD V.—For *negative* lenses. Focus the telescope on the image of a distant object formed by the lens, adjusting for no parallax between the image and the cross-hairs. Record the position of the lens. Remove the lens from the bench. Place on the bench the object diaphragm, adjusting its position so there is no parallax between its image and the cross-hairs, and record its position. Repeat this procedure four times. From the four sets of measurements, compute f for the negative lens.

Show by a carefully drawn diagram how f is obtained by this method.

Note.—For the measurement of the focal length of a photographic lens mounted in a camera, see procedure suggested in Derr's "Photography for Students of Physics and Chemistry," page 15.

EXERCISE 21

THE COMPOUND MICROSCOPE

Object.—The aim of this exercise is to introduce the student to the principle and adjustments of the compound microscope (1) by determining the magnifying power and its relation to (a) the focal length of the objective, (b) the distance of the comparison scale from the eye, (c) the tube length, (d) the focal length of the eyepiece; (2) by calibrating an eyepiece micrometer and measuring two distances by means of it; (3) by determining the index of refraction of a glass plate with reference to air.

General Discussion.—The compound microscope consists essentially of a brass tube or body so attached to a heavy stand that it can be set at an angle or moved up or down. In its lower end the objectives are screwed either directly or to a double or triple nosepiece which facilitates the changing of the objectives, while the eyepieces slide freely into the upper end. The objectives are made of a combination of lenses, by means of which is obtained freedom from spherical and chromatic aberration when used with a Huyghens' eyepiece. This eyepiece is of the form known as a negative eyepiece, and consists of two plano-convex lenses with their plane surfaces turned away from the object. Fixed to the lower part of the stand is the stage on which the object is laid and kept in place by spring clips. Under the stage is an iris diaphragm similar to those used in cameras, by means of which the size of the field of view may be regulated. Below this is a mirror, plane on one side and concave on the other, by which the light may be concentrated upon the object.

In order to focus the instrument the tube containing the objective and eyepiece must be raised or lowered. There are two adjustments to affect this; first, the coarse adjustment to move it rapidly by means of a rack and pinion, and second, a fine adjustment for getting the exact focus. This latter adjustment is made in a variety of ways. In the instrument used in the laboratory a micrometer screw acts by means of a lever arrangement upon the frame carrying the rack and pinion.

Whatever the adjustments of the various instruments may be,

the method of focusing should always be the same; i. e., the tube should be lowered until the objective just clears the object, and then, with the eye at the instrument, the tube should be gradually *drawn up* by means of the coarse adjustment until the object is distinctly visible, after which the fine adjustment should be used to obtain a sharp focus. A drawing showing the construction and optical principles of the compound microscope will be found on the wall of the physical laboratory.

Magnifying Power.—The magnifying power is taken to be the ratio of the angle subtended at the eye by the image as seen in the instrument to the angle subtended at the eye by the object when placed at the distance of most distinct vision, 25 cm., the instrument being focused so that the image is also at the distance of most distinct vision.

The magnifying power is most easily and accurately determined by means of a camera-lucida. This may consist of a prism placed over the eyepiece and constructed in such a way that it is possible to look directly through one part of it down into the tube of the instrument, while the rest of the diagonal surface reflects any object which may lie in a plane perpendicular to the optical axis at the center of the prism, near its line of intersection with a plane through the axis of the instrument and perpendicular to the surface of the prism.

After fastening the instrument to the table it should be tilted so that the line perpendicular to the optical axis of the instrument at the center of the camera-lucida prism and reaching from it to the table is 25 cm. in length. If a scale is now placed at the point where this line strikes the table, it will be seen transposed upon the image of the stage micrometer upon which the instrument is focused. Suppose the scale and the stage micrometer are similarly graduated and that X divisions of the magnified object cover Y divisions of the scale seen directly, then the magnifying power is Y/X .

Determination of the Index of Refraction.—If a ray of monochromatic light falls on a medium of greater optical density at an angle of incidence i , the angle of refraction being r , then the ratio $\frac{\sin i}{\sin r}$ is constant for all values of i . This constant is called the index of refraction for light of the given refrangibility. Let us suppose the first medium to be air. Then it is not difficult

to determine by optical means the value of the angle i , but r cannot be determined with any real approach to accuracy. The determination of μ , the refractive index, is therefore generally effected by indirect means. When the material is in the form of a prism or can be made in that form, the spectrometer is the best instrument to use (see Ex. 17). When the substance, however, is in the form of a thin slab or plate, the compound microscope may be employed for the determination of an average value of μ to three significant figures.

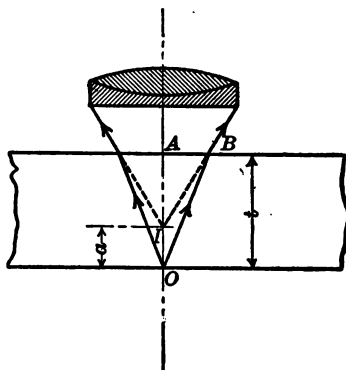


FIG. 18.

Let O (Fig. 18) be a point in a medium of refractive index μ referred to air, and let a small pencil of rays diverging from this point fall directly on the plane upper bounding surface of the medium and emerge into the air in the direction shown.

If A is the point at which the axis of the pencil emerges, and I a point on OA from which the emergent pencil appears to diverge, it is easy to see that

$$\mu = \frac{\sin BIA}{\sin BOA} = \frac{OB}{IB}$$

For a *small* and *nearly normal* pencil of rays, OB is approximately equal to OA and IB to IA . Therefore, we may write without appreciable error

$$\mu = \frac{OA}{IA} = \frac{t}{t - a}$$

where t represents the thickness of the glass and a the distance which the microscope has been moved to cause the object O to appear at I . The determination of μ involves, then, the measurement of t and a .

Apparatus.—Bausch and Lomb microscope complete with suitable objectives and eyepieces, camera-lucida, stage-micrometer and slides.

Manipulation and Computation.—Fasten the microscope to the table and set the tube in an inclined position. In handling the

instrument great care should be exercised against damaging it in any way. The minute lens surfaces of the high power objectives may be very easily scratched by contact with other objects. Remove the eyepiece and fasten the camera-lucida to the upper end of the tube so that the hinge projects to the right. Swing the camera-lucida back, replace the eyepiece $\times 5$ (magnifies 5 diameters) in position and snap the objective of 32 mm. focal length into place by turning the nosepiece. Adjust the mirror until a bright circle of light is visible on looking into the microscope.

Now place a stage micrometer on the stage and focus the instrument until the scale is distinctly visible. To obtain a direct image it will often be necessary to adjust by means of mirror and diaphragm the amount and direction of the light on the object. Oblique illumination often renders the fine scratches more easily visible. The instrument is now in adjustment and ready for use.

The results submitted in the report on this exercise must be the average in each case of at least four (4) *independent settings*.

1. Determine the magnifying power, using objectives of 32 mm. and 8 mm. focal length and a tube length of 160 mm., with eyepiece $\times 5$.

2. Repeat measurements with objective of 32 mm. focal length using 15 cm. distance in place of 25 cm. between the scale and the camera-lucida.

3. Using the objective of 8 mm. focal length, determine the magnification with tube lengths of 140 mm. and also 190 mm., the distance from prism to scale being 25 cm.

4. With objective 8 mm. focal length and tube length of 160 mm., determine the magnifying power using eyepiece $\times 10$ in place of $\times 5$.

5. Using objective of 8 mm. focal length, eyepiece $\times 10$ supplied with micrometer disk and tube length of 160 mm., find by means of the stage micrometer the value of one numbered division of the eyepiece micrometer, i. e., the calibration constant of the eyepiece micrometer, then measure by means of the eyepiece micrometer the distances between the three lines on a glass strip that will be provided.

6. For this part of the exercise a plate of glass is supplied whose index of refraction is to be determined. In doing this use objective of 8 mm. focal length and eyepiece $\times 10$, the tube length

being 160 mm. Set the microscope in a *vertical* position and focus on the glass plate with the fine scratches. Record to the nearest division the position of the head of the fine adjusting screw. Repeat the operation four times so that a fair average may be obtained.

Carefully lay upon the plate with the fine scratches the plate of glass whose index of refraction is to be determined. In order that the scratches may now be seen distinctly, it will be necessary for the body of the microscope to be raised until the objective is the same distance from *I* as it was originally from *O* (see Fig. 18). This is accomplished by refocusing the microscope on the fine scratches, using only the fine adjustment. The distance passed through by the objective will be the distance *OI*. Measure *OA*, the thickness of the plate, with the micrometer caliper.

Knowing that one turn of the fine adjustment screw raises or lowers the instrument .025 cm., determine the distance *OI*, and subtracting this quantity from *OA*, obtain a value for *IA*. With these data compute the mean index of refraction μ for the glass plate.

EXERCISE 22

AIR THERMOMETRY

Object.—This exercise illustrates the use of the air thermometer for temperature measurements and for the determination of the coefficient of expansion of a gas.

General Discussion.—The relation of pressure, volume and temperature for the less easily condensed gases, such as hydrogen, helium, oxygen, nitrogen and air, may be very closely represented at ordinary temperatures by the relation

$$pv = p_0v_0 (1 + \alpha t)$$

where p_0 and v_0 represent pressure and volume of the given mass of the gas at 0°C. , p and v the corresponding pressure and volume of the same mass of gas at any temperature $t^\circ \text{C.}$, and α the coefficient of expansion of the gas. If the volume of the gas is maintained constant, i. e., for $v = v_0$, then

$$p = p_0 (1 + \alpha t)$$

or

$$t = \frac{p - p_0}{\alpha p_0} \quad (1)$$

from which t may be computed when p , p_0 and the coefficient of expansion α of the gas are known. For dry air $\alpha = 0.00367$.

This simple relation for t is not applicable in this exercise because the volume of the gas does not remain constant. The volume of the glass bulb increases when heated from 0°C. to $t^\circ \text{C.}$, and the volume of the exposed portion of the air in C and in the capillary is subject to variation. A formula that is directly applicable can be derived as follows:

Consider the volume of the gas made up of two parts, one of which (the larger part in the bulb) is exposed to the temperature being measured, and the other (the smaller part in C) to the room temperature. Let

V = volume of the bulb at 0°C.

v = volume of the bulb at any temperature $t^\circ \text{C.}$

v' = volume of the exposed stem.

t_1, t_2 = the temperature of the mercury columns at the time of taking the ice and steam readings, respectively. These should differ but slightly throughout the experiment.

R = gas constant per unit mass.

T = absolute temperature.

$K = 0.000027$, the mean coefficient of cubical expansion of the glass per degree C.

By the general gas law, when the bulb is at 0°C . and the exposed air at $t_1^\circ\text{C}$.,

$$p_0V = MRT_0 \quad \text{and} \quad p_0v' = mRT_1$$

When the bulb is at $t^\circ\text{C}$. and the exposed stem at $t_2^\circ\text{C}$.,

$$pv = M'RT \quad \text{and} \quad pv' = m'RT_2$$

Since the total mass of the gas remains constant throughout the experiment,

$$M + m = M' + m'$$

and, therefore,

$$\frac{p_0V}{RT_0} + \frac{p_0v'}{RT_1} = \frac{pv}{RT} + \frac{pv'}{RT_2}$$

Multiplying through by RTT_0 and transposing,

$$p_0VT = pvT_0 + TT_0v' \left(\frac{p}{T_2} - \frac{p_0}{T_1} \right) \quad (2)$$

Bearing in mind that

$$v = V(1 + kt)$$

$$T = \frac{1}{\alpha} + t$$

and that t_1 and t_2 differ so little that they may be taken equal without introducing an appreciable error, equation (2) reduces to

$$\begin{aligned} p_0(1 + \alpha t) = p(1 + kt) + \frac{v'}{V} \cdot \frac{1}{1 + \alpha t_1} \cdot (p - p_0) \\ + \frac{v'}{V} \cdot \frac{1}{1 + \alpha t_1} \cdot \alpha t \cdot (p - p_0) \end{aligned}$$

or

$$t[\alpha p_0 - kp] \left[1 - \frac{v'}{V} \cdot \frac{\alpha}{1 + \alpha t_1} \cdot \frac{p - p_0}{\alpha p_0 - kp} \right] \\ = (p - p_0) \left[1 + \frac{v'}{V} \cdot \frac{1}{1 + \alpha t_1} \right] \quad (3)$$

Neglecting kp in comparison with αp_0 in the *multiplier* of $\frac{v'}{V}$, and solving for t gives

$$t = \frac{(p - p_0) \left[1 + \frac{v'}{V} \cdot \frac{1}{1 + \alpha t_1} \right]}{(\alpha p_0 - kp) \left[1 - \frac{v'}{V} \cdot \frac{1}{1 + \alpha t_1} \cdot \frac{p - p_0}{p_0} \right]} \\ = \frac{p - p_0}{\alpha p_0 - kp} \left[1 + \frac{v'}{V} \cdot \frac{1}{1 + \alpha t_1} \cdot \frac{p}{p_0} \right] \quad (4)$$

which at once reduces to equation (1) if the stem exposure correction involving $\frac{v'}{V}$ and the correction for the expansion of the bulb kp are negligible.

If t is known, the above data may be used to determine α , the coefficient of expansion of the gas under constant volume. Inspection of equation (4) will show that it contains α in various places in such a way that the general solution is exceedingly laborious. To avoid this difficulty, it is advisable to solve for α in the term αp_0 outside of the bracket.

$$\alpha = \frac{p - p_0}{t p_0} \left[1 + \frac{v'}{V} \cdot \frac{1}{1 + \alpha t_1} \cdot \frac{p}{p_0} \right] + \frac{kp}{p_0} \\ = \frac{p - p_0}{p_0} \cdot \frac{1}{t} \left[1 + \frac{x}{1 + \alpha t_1} \left(1 + \frac{p - p_0}{p_0} \right) \right] + \frac{kp}{p_0} \quad (5)$$

where $x = \frac{v'}{V}$. This expression may be solved for α by a method of approximations. Since α enters only in a small correction factor in the term $\frac{x}{1 + \alpha t_1}$, it is legitimate to assume an approximate value for it in this term, and to solve for α by equation (5).

If the value thus obtained differs too widely from the assumed

value of α , a second or even a third approximation should be made. One approximation, however, usually suffices.

To aid in the determination of the pressures p and p_0 , let

H_1, H_2 = reduced barometer readings at the time of ice and steam readings, respectively;

h_1, h_2 = the mean manometer pressures, reduced to 0° C., computed from the observations taken at the temperature of ice and steam, respectively.

If D_1 and C_1 are the observed readings of the mercury manometer when the bulb A is in ice, then the value of h_1 is given by

$$h_1 = \frac{D_1 - C_1}{1 + 0.000181t_1}$$

$$= (D_1 - C_1) (1 - 0.000181t_1) \text{ approximately.}$$

A negative value for h_1 means that the pressure of the air at 0° C. is less than one atmosphere. Similarly,

$$h_2 = (D_2 - C_2) (1 - 0.000181t_2) \text{ approximately.}$$

The actual pressures of the enclosed air at the temperatures of 0° C. and t° C. are, therefore,

$$p_0 = H_1 + h_1$$

$$p = H_2 + h_2$$

Apparatus.—The apparatus provided is a constant volume air thermometer (Fig. 19) consisting of a thin cylindrical glass bulb A at the end of a capillary tube B , which is bent in the manner shown in the figure. The other end of the capillary is connected to a glass tube C about 1 cm. in diameter, containing a black glass indicator I in the upper half. At the lower end of this chamber is the stopcock S . A rubber tube is attached at this point and leads to the glass tube D of the same internal diameter as C . Both C and D are fixed to movable slides which can be clamped at any desired height along the vertical support E . A scale graduated in millimeters is attached to the front side of the support E . By means of this scale the height of the mercury columns at C and D may be obtained, care being taken to avoid parallax.

A boiler is also provided. It consists of a double walled vessel of copper in which the vapor of the boiling liquid circulates around the bulb and down between the inner and outer walls,

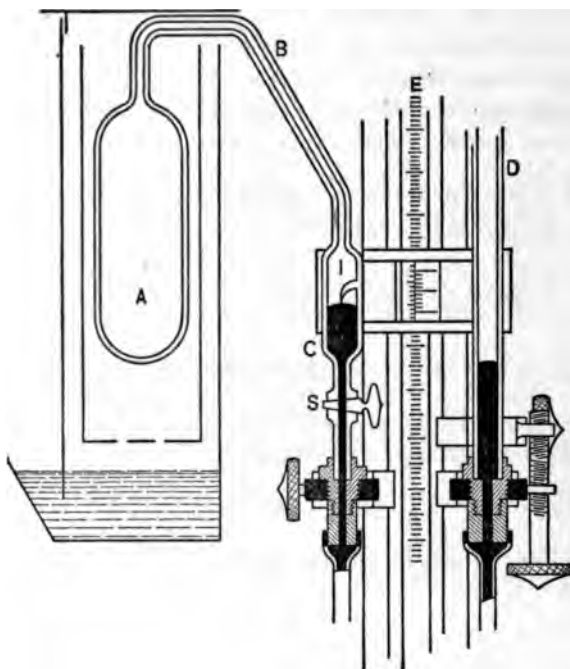


FIG. 19.

thus preventing direct radiation between the outside walls and the bulb of the thermometer.

Manipulation.—The bulb of the thermometer being filled with pure dry air, we must first determine the pressure when the gas occupies a definite reference volume at a known temperature before we can measure an unknown temperature. The known temperature used for this purpose will be the melting point of ice.

To obtain the pressure corresponding to this temperature, proceed as follows:

Crush enough ice to fill the inner cylinder of the heater; cover the wire bottom of the cylinder with about an inch of ice; then lower the thermometer bulb into it until the capillary stem rests upon the bottom of the slot on the side of the heater. During

this operation be *sure* that the *cock S* is closed so as to avoid all possibility of the mercury running over into the bulb. Care must also be taken to exert no pressure on the bulb; otherwise the thermometer is likely to snap off at *C*. Pack the bulb of the thermometer with ice, filling the inner cylinder completely. Place the cover on the heater and allow the apparatus to stand at least ten minutes before taking any readings.

A thermometer should, in the meantime, be attached to the vertical tube *D* with its bulb near the lower part containing the mercury in order to obtain the temperature of the air and mercury, both of which are assumed to be at the same temperature.

Make three independent settings of the meniscus to the indicator and record D_1 and C_1 in each case. Use the mean of the manometer readings in computing the final pressure. Record the temperature t_1 of the thermometer on *D*, also read the barometer and its attached thermometer. As long as the mass of air in the bulb remains unchanged, the reduced pressure p_0 , corresponding to these data, will be the same whenever the air is cooled to 0° C. and the volume is adjusted so that the mercury in *C* stands at the reference mark. For a given thermometer containing a constant amount of gas, this pressure p_0 is a constant. It is always well, however, in practice to redetermine the pressure p_0 corresponding to the ice reading prior to a temperature determination in order to eliminate possible errors arising from leakage or change of volume of the bulb.

Having determined the pressure corresponding to the temperature of 0° C. we may now use this instrument for the determination of an unknown temperature. The unknown temperature used will in this exercise be the boiling point of water under the prevailing conditions. To do this proceed as follows:

Close the cock *S*, light the gas under the boiler, which should be filled with water to a distance of three or four centimeters. Tap water is sufficiently pure for this experiment, although for work of extreme accuracy, distilled water should be used. The ice need not be removed from the heater unless it is desired to save time, as the steam will quickly cause it to melt and run back into the boiler. While waiting for the ice to melt, again record the reading of the barometer and its attached thermometer. When the ice has melted and the water has boiled briskly for five minutes, raise *D* about twenty centimeters and carefully open *S*.

Adjust the height of *D* until the mercury in *C* comes exactly to the reference mark. Wait several minutes to see if the pressure remains constant; if so, record the position of *C* and *D* as before. Readjust two or three times and take the mean of the readings. Record also the reading t_2 of the thermometer.

In order to be able to apply the stem exposure correction, it will be further necessary to obtain a value for $x = v'/V$, which quantity is used in the correction terms of the above formulas. A comparatively rough determination of this ratio suffices to compute the correction with sufficient precision. Let as before

V = volume of bulb

v' = volume of stem

H_2 = reading of barometer

and, in addition,

h_2' = height of the mercury column in *D* above *C* when, with the bulb *A* at steam temperature, *D* has been raised so that the mercury displaces practically all the air in *C*. Care must be taken not to raise it so far up as to produce capillary depression. Then by Boyle's law

$$(H_2 + h_2)(V + v') = (H_2 + h_2')V$$

from which we derive

$$\frac{H_2 + h_2'}{H_2 + h_2} = 1 + \frac{v'}{V}$$

or

$$x = \frac{H_2 + h_2'}{H_2 + h_2} - 1$$

which is the value to be used in the correction term.

On completing the experiment, be sure that the apparatus is left as follows:

The cock *S* should be closed and *C* should be raised so that the bulb is out of the heater. The mercury in *C* should stand two or three centimeters below the reference point to prevent its soiling the inside of the tube where the readings are taken. *D* should now be raised above *C* so that there may be no tendency for air to leak into the apparatus.

Computations.—First. Assuming the volume coefficient of the gas contained in the bulb to be 0.00367, compute, by means of (4), the temperature of the steam at the time of the experiment.

Second. Compute the temperature of boiling from the corrected barometer reading and the formula $t = 100 + \frac{3}{80}(H - 760)$. Using this value for t , compute α by equation (5), making as many approximations as may be necessary to obtain a check within 0.5 per cent. of the assumed value.

EXERCISE 23

COEFFICIENT OF LINEAR EXPANSION

Object.—This exercise illustrates a fairly accurate method for the measurement of the coefficient of linear expansion between room temperature and 100° C. (approximately) of a solid material in the form of a rod of about a meter's length.

General Discussion.—The mean coefficient of linear expansion of a solid between any two temperatures t and t' is the ratio of the average increase of length per degree between these two temperatures to the length at some temperature taken as a standard. If the length at 0° C. be adopted, then this coefficient becomes

$$\alpha = \frac{L_t - L_{t'}}{L_0(t - t')} \quad (1)$$

which is called the zero mean coefficient of linear expansion. If t' is taken equal to 0°, then α becomes the coefficient ordinarily found in tables of physical constants, that is, the mean coefficient of expansion between 0° and t ° C.:

$$\alpha = \frac{L_t - L_0}{L_0 t} \quad (2)$$

In this experiment, the mean coefficient of expansion between room temperature and steam temperature is to be determined. Since the expansion between 0° C. and room temperature of the metal rods used is small, the length measured at room temperature is used instead of L_0 in the denominator of formula (1).

Apparatus.—Two long brass cylinders are arranged concentrically so that when steam is sent into one end of the inner one it flows to its other end, and then back through the outer cylinder, thus surrounding the inner tube with a steam jacketing. The rod whose expansion is to be determined is placed along the axis of the inner tube. The change in its length is measured by means of a micrometer gauge, which may be set up against one end while the other end is against a stop. The extension rod on the gauge and the stop are of invar. The gauge and stop are at-

tached to a rigid iron frame so pivoted that it may be turned back from the heating apparatus except when the actual measurements are being made. Thermometers for measuring the temperature of the bar and a meter scale for measuring its length are also provided.

Manipulation.—First measure at room temperature with the meter scale the lengths of the rods assigned. Handle them as little as possible to avoid raising their temperature above that of other objects in the room. Then place each rod in the cylinder *before the steam is turned on*, and obtain the setting of the micrometer. Before making the setting the bar should remain in the jacket several minutes to acquire the temperature of the jacket. Then tip forward the iron frame, push in the left stop and set the spring to hold it in place. Making sure that the rod is against the left stop, advance the micrometer spindle, using the ratchet, until contact is made with the rod, as indicated by the slipping of the ratchet. Make four independent settings of the micrometer, rotate the bar a quarter revolution by means of the specially devised clip, and take another set of four readings on the micrometer. Continue this procedure for the next two quarter revolutions, thus obtaining in all four sets of four readings each. It is advisable to record these 16 readings in one column. If these readings show no progressive change, thus indicating that the bar is in thermal equilibrium with its surroundings, the temperature of the bar may be read from the thermometers attached to the apparatus. Take two readings on each thermometer.

The other bar should now be placed in the apparatus and readings taken in exactly the same way.

Note that an error of 0.5 millimeter can be readily made by the careless reading of the micrometer.

Do not introduce steam into the apparatus until you are satisfied that the readings on both bars at room temperature have been correctly taken. Time may be saved, however, by beginning to heat the boiler before the readings are completed, making sure that it is not connected to the expansion jacket. In starting the boiler, make sure that it is from $\frac{1}{3}$ to $\frac{1}{2}$ filled with water, and that the pressure has some means of relief. Take care that there is water in the boiler at all times.

During the heating of the bar the frame should be tipped back

so as to prevent its heating. Do not start measurements until the readings on the two thermometers have become stationary. In the meantime read the *barometer and its attached thermometer*.

After the thermometers have become stationary put the measuring frame in place and take a set of four readings on the heated rod. Then draw back the stop and micrometer, tip back the upper supporting frame, turn the bar through 90° , and wait three minutes before taking a second set of readings. Meanwhile record the temperature indicated by the thermometers and record all data necessary to correct for stem exposure.

In the same way obtain a third and fourth set of four micrometer readings and the temperature data. If these micrometer readings indicate by the absence of progressive change that the bar has been in thermal equilibrium with the steam and jacket, proceed to measure the other rod in a similar manner. After introducing the second rod into the jacket, allow it to remain there for at least six minutes before taking any readings. Always keep the steam passing through the jacket during the readings.

Computations.—The change in length of each rod is given by the difference in the average micrometer readings at the two temperatures, if these have been correctly taken.

The indications of the thermometers at the higher temperatures are subject to error due to the exposed stem. The correction, using the centigrade scale, is obtained from

$$t' = t + 0.000156(t - t_a)n$$

where t' is the temperature corrected for stem exposure, t is the temperature as read, t_a is the temperature of the portion of the thermometer exposed to the air, and n is the length (in degrees) of the *mercury thread* thus exposed.

The temperature of the bar may also be taken as identical with the temperature of the steam as found from the barometric pressure by the use of a steam table, or by the following formula:

$$\text{temperature of steam} = 100^\circ + \frac{3}{80}(H - 760)$$

where H is the barometric pressure in millimeters *corrected* for the expansion of mercury and scale. The method for correcting the barometer reading may be found in the experiment on the barometer, or in any good laboratory manual. Compare

the steam temperature thus obtained with that found from the thermometer readings corrected for stem exposure. Use the mean of the two as the temperature of the bar when heated.

Compute the coefficient of expansion of each bar used. From these results compute the coefficient of expansion for each bar in terms of Fahrenheit temperatures.

Problems.—(1) From the sets of micrometer readings on one bar at the higher and lower temperatures compute the average deviation of a single setting at each temperature, and the average deviation of the mean setting at each temperature. What percentage effect would these uncertainties in the micrometer readings produce in the value of the coefficient of expansion?

(2) If the iron frame which supports the stop and micrometer gauge and whose length may be taken as the same as that of the bar (assuming that the gauge and stop both expand the same as the iron frame) increases in temperature 1°C . between the measurements on a bar when cold and those when it is hot, what percentage error would this introduce into the value of the coefficient of expansion as determined? Coefficient of expansion of iron is about 0.000012 referred to centigrade scale.

EXERCISE 24

PRESSURE AND BOILING POINT

Object.—The object of this exercise is to determine the pressure of saturated vapor of water at different temperatures by the dynamical or boiling point method and to obtain the specific volume of the saturated vapor at 100° C.

General Discussion.—A vapor in equilibrium at a definite temperature with the pure liquid is said to be saturated. Any attempt to increase the density and the pressure of the vapor, the temperature remaining constant, will cause condensation without change in the density and in the pressure. Conversely, if the volume occupied by the saturated vapor in contact with its liquid is increased, some of the liquid will evaporate to maintain the density and the pressure of the vapor unchanged. In case the saturated vapor is *not in contact with its liquid*, any increase in the volume of the saturated vapor or reduction in the pressure at a definite temperature will change the saturated vapor into an unsaturated vapor. Likewise an increase in the temperature of a saturated vapor, *not in contact with its liquid*, will convert the saturated vapor into an unsaturated one.

Saturated vapors do not follow the general gas law

$$\frac{pv}{p_1v_1} = \frac{T}{T_1}.$$

The density and the pressure of such vapors do, however, increase with rise in temperature.

There are two methods for the experimental determination of the pressure of a saturated vapor at various temperatures, called the statical and the dynamical methods, respectively. In the statical method the liquid is introduced into the top of a barometer maintained at a constant temperature and the depression of the mercury column is observed. This method is liable to large errors due to impurities in the liquid, especially when these impurities are very volatile.

The dynamical method depends on two facts, first that the pressure of a saturated vapor increases with rise in temperature,

and second that a liquid will boil when the pressure of its saturated vapor equals the external pressure on the liquid. As a consequence, on heating a liquid, its temperature will steadily rise until the temperature is reached for which the saturated vapor pressure of the liquid equals the external pressure. Boiling will continue at this constant temperature, provided the external pressure does not change, until all of the liquid is converted into vapor. Since this constant temperature and external pressure can be measured with precision, the boiling point of a pure liquid under different external pressures affords an excellent method for the measurement of the pressure of a saturated vapor at different known temperatures. The presence of impurities has but slight effect in this method in comparison with the statical method.

An important thermodynamic relation exists between the external pressure p and the temperature of boiling t of the liquid, i. e., between the temperature t and pressure p of its saturated vapor, namely

$$\frac{dp}{dt} = \frac{L}{T(V_2 - V_1)} \quad (1)$$

where L is the heat of vaporization of the liquid at any temperature $t^\circ \text{C.}$, T the absolute temperature corresponding to $t^\circ \text{C.}$, and V_1 and V_2 the specific volumes of the liquid and its saturated vapor at the temperature $t^\circ \text{C.}$, respectively. In this expression L , V_1 and V_2 are functions of the temperature. For *water* Clausius deduced from Regnault's measurements the following empirical formula for L :

$$L = 607 - 0.708 t$$

A more recent equation expresses more closely the experimental values obtained for *water*, giving 539.1 calories per gram for the heat of vaporization at 100°C. under normal pressure. It is

$$L = 539.1 - 0.655 (t - 100) - 0.00081 (t - 100)^2 \quad (2)$$

While the direct experimental determination of the specific volume of a saturated vapor at any temperature t with accuracy is difficult, it may be computed from equation (1) provided $\frac{dp}{dt}$ and L are known, since V_1 for a liquid in cubic centimeters per gram is approximately equal to unity and is usually negligible

with respect to V_2 in any computation. Assuming a knowledge of L , it becomes necessary to determine $\frac{dp}{dt}$ experimentally in order to compute the specific volume V_2 of the saturated vapor.

Reference to any work on thermodynamics will also show that equation (1) holds likewise for the change of state from solid to liquid, as for example from ice to water. In this change of state the letters have the following meaning: L denotes the heat of fusion, T the melting point on the absolute scale, V_1 and V_2 the specific volumes of the solid and the liquid at the temperature T .

References: Duff, pages 240–252; also Edser, *Heat*, Chap. XVII, on Applications of Carnot's Theorem.

Apparatus.—The apparatus consists essentially of the boiler A supplied with an accurate thermometer B , condenser C , air reservoir D , and open manometer E .

Manipulation.—The student is cautioned to follow the directions closely, as otherwise the apparatus may easily be damaged.

First remove the plug G (Fig. 20) and ascertain whether the boiler is about half full of water by means of the glass gauge tube. If this is not the case, add the proper amount and replace G . Second, start the Bunsen aspirator and open the pinchcock J and the stopcock H , being sure that the pinchcock I is firmly closed. When the pressure has been reduced to 5 to 7 cm. of mercury, or to the limit of the pump if the pump is not able to reduce the pressure so far, the stopcock H should be closed and then the pinchcock I opened before the aspirator is turned off. The reading of the manometer now remaining constant indicates that the apparatus is air tight. If this is not the case, notify an instructor. Third, turn the water through the condenser C so as to have a good stream flowing through, and light the burner under the boiler. A small flame not greater than 3 cm. in height will be sufficient to produce boiling at this reduced pressure. When the reading of the thermometer B becomes constant, record this temperature and the reading of both branches of the manometer; record the reading of the thermometer F attached to the manometer, also the height of the barometer and its temperature. This completes all data necessary for the first set of observations.

Next allow air to enter the apparatus through H until the

manometer indicates an *increase* of pressure of about 6 to 10 cm. This increase will immediately cause the boiling to cease until the temperature has risen to the new boiling point. When this constant temperature has been reached it should be recorded together with the manometer readings as before. Determine

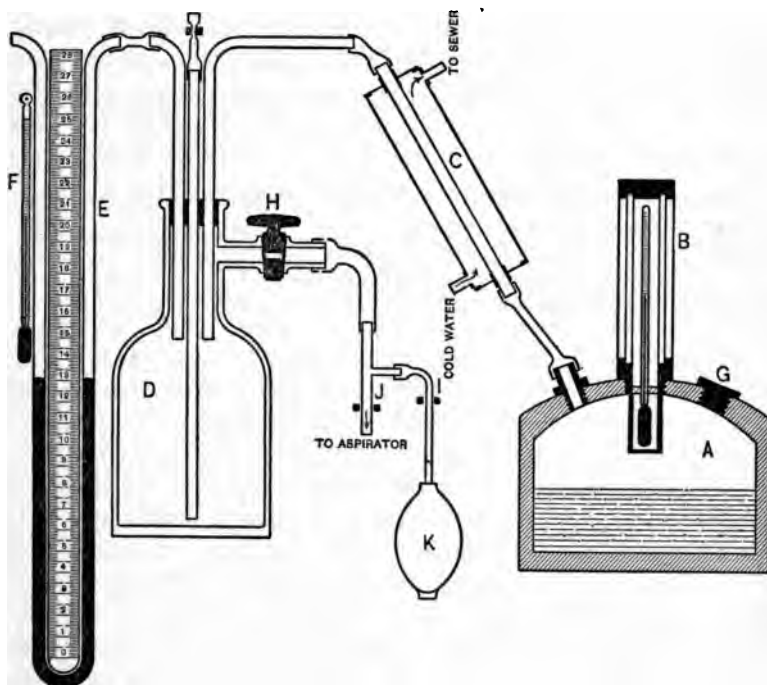


FIG. 20.

in this way the boiling points corresponding to seven or eight different pressures. Take the last observations at a pressure of 3 to 5 cm. above atmospheric pressure by forcing air into the apparatus by means of the bulb *K*. Do not increase the pressure above this point under any condition.

Computations.—Thermometer *B* may be considered to read temperatures correctly. Reduce the manometer readings to 0°C ., assuming the manometer scale to be correct at room temperature. Plot the reduced data, giving the *total* pressure in millimeters of mercury as ordinates, and temperature as abscissas. This will show graphically the relation between the

pressure and boiling point of the liquid, or, in other words, the variation of the saturated vapor pressure of the liquid with the temperature.

Next obtain the value $\frac{dp}{dt}$ at 100° C. from the plot by drawing the tangent to the curve at this point.

With this value and that of L computed by means of equation (2), calculate by equation (1) the specific volume of saturated steam, v_2 at 100° C., remembering that both $\frac{dp}{dt}$ and L must be expressed in mechanical units.

Problem.—Compute from Formula (1) the melting point of ice under a pressure of fifteen atmospheres.

Given: heat of fusion of ice 80 calories per gram

specific volume of ice 1.09 c.c. per gram.

specific volume of water at 0° C. 1.00 c.c. per gram.

EXERCISE 25

HYGROMETRY

Object.—The object of this exercise is to determine the hygrometric state of the atmosphere by the use of an Alluard Dew-Point Hygrometer and Wet and Dry Bulb Hygrometers. From the readings of these instruments to obtain directly from tables: (1) the dew-point, (2) the pressure of the water vapor in the air, (3) the relative humidity, (4) the absolute humidity.

General Discussion.—The atmosphere always contains water vapor due to evaporation. This water vapor is usually in an unsaturated state and exerts a pressure p , called the aqueous vapor pressure. At the temperature of the atmosphere saturated water vapor exerts a pressure, P , which, according to Dalton, is independent of the presence and the pressure of the air itself.

Dew-Point.—The dew-point is the temperature at which the moisture in the air would begin to precipitate. It is, also, the temperature to which the atmosphere must be cooled in order that the water vapor which is present may be saturated. At the dew-point the saturated vapor pressure of water equals the original pressure p of the aqueous vapor in the atmosphere.

Absolute Humidity.—The absolute humidity is the actual mass of water which the air contains as vapor in a unit volume. It is usually expressed in grams per cubic meter or in grains per cubic foot. The absolute humidity is merely another name for the density of water vapor present in the atmosphere.

Relative Humidity.—Relative humidity or fraction of saturation is the ratio of vapor (m) actually present in a unit volume of the atmosphere to the mass (M) for saturation at the same temperature, $\text{r.h.} = \frac{m}{M}$. This is equivalent to the statement that the relative humidity is the ratio of the pressure (p) of the water vapor in the atmosphere at the time of the experiment to the pressure that saturated water vapor (P) would exert at the same temperature, i. e., $\text{r.h.} = \frac{p}{P}$.

The Regnault Hygrometer.—For a description of the apparatus,

see U. S. Department of Agriculture, Psychrometric Tables, pp. 7-8. The use of this instrument rests on the assumption that the saturated aqueous vapor pressure at the dew-point is equal to the actual pressure (p) of the aqueous vapor in the air at the time of the experiment. If, then, the dew-point is experimentally determined, the saturated water vapor pressure at the dew-point, as found in a table of saturated water vapor pressures, is the desired pressure p .

From the table of saturated vapor pressures of water the saturated vapor pressure P at the temperature of the atmosphere may be obtained and the relative humidity computed by ratio $\frac{p}{P}$.

Wet and Dry Bulb Psychrometer.—This apparatus consists of two sensitive thermometers mounted side by side. One thermometer bulb is wrapped in muslin which is kept moist. On placing these thermometers in a current of air, the thermometer with the bare bulb will indicate the temperature of the air, while the other will indicate a lower temperature. The difference in temperature is due to evaporation at the surface of the wet bulb and depends upon the amount of water vapor present in the air, i. e., upon the saturation of the air. The drier the air the more rapid will be the evaporation and the greater the difference in the readings of the wet and dry bulb thermometers. If the air contains saturated water vapor, the two thermometers will read the same.

The simplest formula which has been found to give the relation of vapor pressure p at the temperature t of the air to the saturated vapor pressure p_1 at the temperature t_1 of the wet bulb and to the difference in temperature of the wet and dry bulb thermometer is:

$$p = p_1 - AH(t - t_1) \quad (1)$$

where H is the barometer reading and A is a constant depending on the form and size of the wet bulb thermometer and the velocity of ventilation. By using a ventilating velocity of ten feet or more per second, i. e., if the air in the neighborhood of the wet bulb moves at the above rate, a constant maximum depression of temperature is reached, and the form of the bulb is eliminated. Under these conditions the various instruments will give the same reading of wet bulb, and the same hygrometric tables will apply to all.

The results of many observations have shown that equation (1) in metric units, pressure in *millimeters* and temperature in degrees centigrade, is best represented by

$$p = p_1 - 0.000660H(t - t_1) (1 + 0.00115t_1) \quad (2)$$

where $A = 0.000660(1 + 0.00115t_1)$.

Expressed in English units, p and H in *inches* and temperatures Fahrenheit, equation (3) becomes

$$p = p_1 - 0.000367H(t - t_1) (1 + 0.00064t_1) \quad (3)$$

It is clear that the pressure of the water vapor in the atmosphere (p) may be computed by direct observation of t , t_1 and the barometer height H . Then from the table of saturated water vapor pressure, the saturated vapor pressure P at temperature t may be found, and the relative humidity computed by the formula

$$\text{r.h.} = \frac{p}{P}$$

In the Smithsonian Meteorological Tables and in the U. S. Psychrometric Tables the values of relative humidity and temperatures of dew-points are given for any barometric pressure in terms of t and $t - t_1$. These tables have been computed by means of formulas (2) and (3).

For a more complete treatment of the subject of hygrometry the student is referred to: U. S. Department of Agriculture, Psychrometric Tables, pp. 5-14; Glazebrook, Heat, pp. 146-151; Millikan, Mechanics, Molecular Physics and Heat, pp. 164-170; and to his own lecture notes on the subject of heat.

Apparatus.—The apparatus consists of an Alluard dew-point hygrometer, also two wet and dry bulb psychrometers, giving readings in centigrade and Fahrenheit degrees, respectively, and a hygrodeik.

Manipulation: PART I.—Use of Alluard hygrometer. Following the procedure outlined on page 7 of the U. S. Department of Agriculture circular, determine the dew-point, using the Alluard hygrometer, by taking the average of the temperatures at which dew appears and disappears. Never cool apparatus more than two degrees below dew-point as obtained by a preliminary experiment. Make at least five such determinations and use their

average as the dew-point. Record each time the temperature of the room by the second thermometer.

PART II.—Use of wet and dry bulb hygrometer. (1) Use the sling psychrometer following the directions on page 6 of the U. S. Department of Agriculture Psychrometric Tables. Obtain three independent values for t and t_1 .

(2) Swing the second type of wet and dry bulb psychrometer to and fro with pendular motion to insure a circulation of air of at least ten feet per second. Continue this motion until the temperature by the wet bulb becomes constant and a minimum. Record this reading and immediately afterward that of the dry bulb thermometer. Obtain three independent values for t and t_1 .

PART III.—Determine with the hygrodeik the relative humidity, the absolute humidity and the dew-point.

Computations: PART I.—Using the average dew-point and the Smithsonian Meteorological Tables, obtain (1) vapor pressure in millimeters by Tables 36 or 37, (2) relative humidity by Table 45, (3) the absolute humidity (m) using value of relative humidity and Table 39.

The absolute humidity (m) in grams per cubic meter is given by:

$$m = 1293 \times 0.622 \times \frac{1}{1 + 0.00366t} \times \frac{p}{760} \quad (4)$$

where p = pressure in millimeters of the water vapor in the atmosphere at the temperature t° C., 1293 is the density of air in grams per cubic meter at 0° C. and 760 mm., 0.622 the density of water vapor compared with air under like conditions of temperature and pressure, and 0.00366 is the coefficient of expansion of air.

Compute m by formula (4), using the value of p already found from the tables, and compare it with the value obtained from the Meteorological Tables.

It is evident that if m be determined by direct measurement, p may be computed, and then the dew-point and relative humidity may be obtained from proper tables. The *Chemical Hygrometer* gives m by direct measurement.

PART II. (a).—Using the mean value of t and t_1 as obtained by the centigrade psychrometer and with the barometer reading H , compute the value of p by equation (2). With this value and

Tables 36 or 37 of the Smithsonian Meteorological Tables compute the relative humidity and find the dew-point. Check the value of relative humidity by use of Table 45.

(b) With the average value of t_1 and t in degrees F. obtain the following hygrometric quantities, using the U. S. Psychrometric Tables: (1) dew-point, (2) vapor pressure, (3) relative humidity, (4) mass of vapor in grains per cubic foot.

Problems.—(1) Assuming unsaturated water vapor to obey the fundamental law of gases up to the point of saturation, prove the correctness of the relationship:

$$\text{r.h.} = \frac{p}{P} = \frac{m}{M}$$

(2) Prove that the absolute humidity (m) in grams per cubic meter is given by

$$m = 1293 \times 0.622 \times \frac{1}{1 + .00366t} \times \frac{p}{760}$$

EXERCISE 26a

SPECIFIC HEAT

Object.—(1) To bring out the fundamental principles of calorimetry through the measurement of the specific heat of a solid by the Method of Mixtures. (2) To illustrate one method of making the radiation correction, the Regnault-Pfaundler method. (3) To gain experience in the correct use of a calibrated mercurial thermometer having an accuracy better than 0.1°C .

It is possible, if the method of procedure given is carefully followed, to obtain a value for the specific heat of the solid under test reproducible to 0.5 per cent.

General Discussion.—Read Duff, pages 226–234 (fourth edition), also Edser's *Heat for Advanced Students*. An excellent discussion of thermometry will be found in the Bureau of Standards Circular No. 8, entitled *Testing of Thermometers*. Other references: Goodwin's *Physical Laboratory Experiments* and Watson's *A Text-Book of Practical Physics*.

A calorimeter is an apparatus for the measurement of quantities of heat. It consists, usually, of a cylindrical polished nickel-plated copper vessel, containing water or suitable liquid, which rests on an insulating support inside of a nickel-plated vessel. The outer vessel or shield may be exposed to the air, lagged with felt or other heat-insulating material, or it may be constructed to hold a mass of water so as to ensure a constant temperature of surroundings throughout the experiment. In some cases the calorimeter is provided with a cover to lessen evaporation. The air space between the calorimeter and shield is small to guard against convection currents. A stirrer is provided to maintain the calorimeter and its contents at a uniform temperature.

If energy supplied to the calorimeter be transformed into heat, it will cause a rise in the temperature of the calorimeter and its contents. Provided no heat can escape from the calorimeter nor be received by the calorimeter from the surroundings, it follows from the conservation of energy that the heat absorbed by the calorimeter and contents must equal the quantity of energy supplied.

A vacuum-jacketed calorimeter is sometimes employed to lessen the radiation error, and to eliminate loss or gain of energy by the calorimeter through conduction and convection.

The heat quantities most frequently measured are the specific heats of solids, liquids and gases, the heats of fusion and vaporization, the heat value of fuels and the mechanical equivalent of heat.

Probably the largest source of error in most calorimetric work arises in the determination of the change in temperature of the calorimeter and contents, which is usually not more than 3° to 5° and may be less. To ensure even a reasonable precision, mercury thermometers sensitive to 0.01° C. are required. According to the U. S. Bureau of Standards, the highest accuracy attainable with mercurial thermometers of the Beckmann type, where a temperature change of a few degrees is to be measured, after most careful standardization is hardly better than 1 or 2 parts in 1,000 of the measured interval, and with a suitable platinum resistance thermometer an accuracy of about 1 or 2 parts in 10,000 may be attained only by the use of elaborate and expensive apparatus.

An important source of error that must be considered in work where *high accuracy* is desired is the variation in the specific heat of water with the temperature. Experiment has shown that the heat required to raise the mass of one gram of water 1° C. is not constant. As a consequence several different calories have been suggested, among them the following:

1. The mean calorie, defined as one hundredth part of the heat required to raise one gram of water from 0° C. to 100° C.
2. The 15° calorie, or the heat required to raise one gram of water from 14.5° to 15.5° C.
3. One fifth of the heat required to raise one gram of water from 15° to 20° C.
4. The 20° C. calorie, or the heat required to raise one gram of water from 19.5° to 20.5° C.

The following table of Callendar and Barnes gives the specific heat of water at different temperatures in terms of the 20° calorie.

Temp.	20° Calorie	Joules
0° C.	1.0094	
5	1.0054	4.2130
10	1.0027	4.1999
15	1.0011	4.1912
20	1.0000	4.1851
25	0.9992	4.1805
30	0.9987	4.1780
40	0.9982	4.1769
50	0.9987	4.1785
90	1.0053	4.2014

In a calorimetric process where the energy is supplied at a *constant* rate, such as in the determination of the heat of vaporization, mechanical equivalent of heat and the specific heat of a gas under constant pressure, the errors due to conduction, convection and radiation may be largely eliminated by starting the experiment below room temperature and cutting off the energy supply when the temperature of the calorimeter and contents reaches the same number of degrees above room-temperature. This is known as the Rumford method, and it rests on the assumption that the heat absorbed from the room by the calorimeter while colder than the shield equals the heat lost from the calorimeter to the room while its temperature is above the shield. In a discontinuous calorimetric process, where the energy is introduced in one installment, as in the specific heat of solids and liquids by the method of mixtures, the Rumford method is not directly applicable. However, even in this case the errors due to conduction, convection and radiation will be lessened by adjusting the initial temperature of the calorimeter below the shield temperature so that the rise of temperature on introducing the hot body will carry the temperature of the calorimeter and contents to approximately the same amount above the shield temperature.

The chief sources of error in this exercise are:

1. Loss of heat in the transfer of the hot body.
2. Errors of thermometry.
3. Error in the determination of the water equivalent of the calorimeter, stirrer and thermometer.
4. Errors arising from conduction, convection, radiation and evaporation.
5. Error due to the change in the specific heat of water with the temperature.

Apparatus.—The apparatus consists of an open calorimeter of polished nickel-plated copper which rests on an insulating support inside a similarly nickel-plated vessel or shield. The calorimeter thermometer graduated to 0.1° C. is held in place by a support fixed to the shield base. The calorimeter is protected from the heating apparatus by a double-walled screen. The heating apparatus includes a boiler and the heater proper. The latter consists of two concentric brass cylinders joined at the top and bottom. The inner cylinder is closed at the top and bottom by

corks, thus forming a receptacle in which to heat the substance. The outer cylinder is heavily lagged with felt to prevent loss of heat and to facilitate the handling of the heater while hot. Two tubes lead through the outer cylinder to the space between the cylinders. The upper tube is connected to the boiler by a rubber connection so that steam passes around the inner cylinder and escapes to the condenser through the lower tube.

The substance to be heated is placed in the inner cylinder, which is then closed at the top by the cork through which the thermometer passes. As a large portion of this thermometer extends above the cork, an auxiliary thermometer reading in degrees centigrade should be used to determine the temperature of the exposed stem. A fourth thermometer, also graduated in degrees centigrade, will be required to record the temperature of the shield surrounding the calorimeter and should be placed so as to touch the inside surface of the shield. The corrections for all thermometers used with the apparatus will be given by the instructors.

Manipulation.—Weigh the substance to 0.1 gram. Place it in the heater and begin heating it at once, as considerable time is required to raise the substance to a uniformly high temperature. When placing the substance in the heater, allow it to rest lightly on the lower cork, making sure that the string to which it is attached is held taut by the top cork. Place the thermometer that is to record the temperature of the substance in a hole that will be found in the top of the cylindrical block. While the substance is heating, weigh the dry calorimeter and the stirrer *without handle*. Next fill the calorimeter with water to about two thirds of its capacity and weigh. Ordinary tap water may be used.

All weighings in this exercise should be made to 0.1 gram.

The metal used is of such form that the equalization of heat between it and the calorimeter is very rapid. Since the relative masses of metal, calorimeter and water are chosen to give a rise in temperature of 4° to 6° centigrade, it is advisable to have the temperature of the water from two to three degrees below shield temperature when the metal is introduced in order to reduce the radiation correction to a minimum.

After weighing and adjusting the temperature of the calorimeter, it should be placed within the shield, and then the

calorimeter and shield thermometers should be so arranged that both can be read by the observer *A*. These thermometers should be so placed that they will not be in the way of the heater when it is swung over the calorimeter for the introduction of the substance.

During the heating of the substance, it is advisable to go through the operation of introducing it into the calorimeter to become familiar with the procedure and to guard against mistakes and loss of time in the actual experiment. Of course in these *preliminary trials the substance should never be actually placed in the water of the calorimeter.*

A blank form for recording data should next be prepared with one column for the *time given by the watch in hours, minutes and seconds*; a second for the corresponding calorimeter temperatures; a third for the calorimeter thermometer corrections; a fourth for the temperature of the substance; a fifth for the temperature of the shield; and a sixth for the readings of the thermometer attached to the stem of thermometer in heater. The times *should be written down beforehand* in one-minute intervals for a period of nine minutes, then in one-half-minute intervals for a period of eight minutes or more. A record should also be made of the number of exposed divisions of the thermometer in the heater. Such a carefully prepared data sheet is essential to the successful performance of this experiment.

When the *temperature of the substance in the heater becomes constant*, as shown by two minute readings taken for a period of ten minutes, begin to take readings of all four thermometers every minute for eight minutes. The observer *A* should read the temperature of the calorimeter (to 0.01°C.), the temperature of the shield (to 0.1°C.), and should *record all readings*. From the time that *the first reading is made until the end* of the experiment, the observer *B* at the calorimeter should continue to *stir the water* with a regular up-and-down motion. This observer should also read the temperature of the hot solid, the temperature of this thermometer's exposed stem, and should give the signals for calorimeter readings.

At the end of nine minutes and thirty seconds the observer *A* should swing the heater over the calorimeter, remove the lower cork, and be ready to remove the top cork and to lower the substance by means of the string quickly but carefully into the

calorimeter at the instant the signal for the *tenth minute* is given. The hot body should be so placed that it does not strike the bulb of the calorimeter thermometer, not only to prevent accidents, but also to avoid local heating of water in the neighborhood of the bulb. No attempt should be made to read the calorimeter thermometer at the time the substance is introduced, as this reading may be deduced from those previously recorded. It should, however, be read one half minute after the substance is introduced, then every half minute for four and one half minutes, and finally every minute for eight minutes. Record also at these intervals the temperature of the calorimeter shield. It is absolutely essential that water in the calorimeter be stirred thoroughly and regularly throughout this entire experiment, especially when the substance is introduced and during the following five minutes.

During the whole experiment the temperature of the shield should remain practically constant. If it changes more than two degrees, this fact should be reported to an instructor.

From these data estimate as nearly as possible the maximum rise in temperature of the water and compute the specific heat of the substance, assuming the specific heat of the calorimeter and stirrer to be 0.093. In this computation corrections should be applied to thermometer readings for standardization and exposed stem whenever necessary.

Show the results to the instructor to ascertain whether they are sufficiently precise. If not, the experiment must be repeated.

EXERCISE 26b

SPECIFIC HEAT

Object.—To correct the data obtained in Exercise 26a for radiation, and to compute with these corrected data the specific heat of the solid.

General Discussion.—Since there is no insulator for heat, all measurements of quantity of heat are very difficult. It is, therefore, desirable to arrange the experiment so as to make the heat losses (or gains) so small that a fairly accurate allowance may be made for them. The losses can generally be made small by having the temperature of the calorimeter differ but little from that of the room. Then an estimate of the amount of loss can be made by a judicious timing of the temperature observations. This estimate is generally called "correction for radiation," the term "radiation" here including, not only radiation proper, but all the causes of loss, such as conduction, convection and evaporation.

In making calorimetric measurements so that the heat lost or gained by radiation may be allowed for or eliminated, it is customary to divide the time covered by the measurements into three periods: a preliminary period, the main period, and an after period.

Preliminary Period (P.P.).—This is the time during which the temperatures of the calorimeter, substance and shield are recorded every minute for at least nine minutes before the heated body is introduced. This period of readings furnishes data for determining the temperature of the calorimeter at the moment the substance is introduced (which in general cannot be read), and also for determining the average rate of gain or loss of heat of the calorimeter at that time.

Main Period (M.P.).—This interval begins with the introduction of the hot body and ends when the change in temperature of the calorimeter becomes uniform.

After Period (A.P.).—This consists of a number of one-half-minute intervals (sixteen to twenty) taken while the change of temperature is uniform, which can then be used to determine the

rate of gain or loss of heat under the conditions prevailing at the end of the experiment.

Temperature-Time Plot.—If the data obtained as described above be plotted with temperatures as ordinates and times as abscissas, curves of the general type shown in Fig. 21 will be obtained.

The sloping line *AB* indicates that the calorimeter is at a lower temperature than its surroundings and is, therefore, absorbing heat. At *B* the hot body is introduced and the curve rises, owing to the heat given out by the hot body, until at the point *C* the rate of loss from the calorimeter equals the rate at which the hot body is giving heat to the calorimeter.

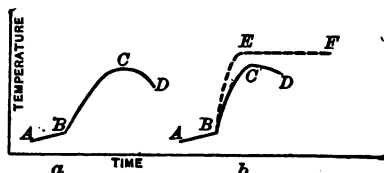


FIG. 21.

From this point the loss of heat to the surroundings begins to exceed the heat emitted by the hot body, and very quickly the calorimeter and heated body, being at a uniform temperature, cool off at a constant rate as is indicated by *CD*.

In specific heat determinations the curve may take any form between curve *a* and curve *b*, according to the rate of equalization of temperature between the hot body and the calorimeter. For a poor conducting material, not in small pieces, a curve like *a* will be obtained. For a metal having large surface, a curve similar to *b* is obtained unless there is no heat lost from the calorimeter. In the latter case the curve becomes *ABEF*.

In order that curves similar to the above types may be obtained, it is absolutely essential that the stirring be so efficient and the thermometer so placed in the calorimeter that the temperature indicated is the mean temperature of the calorimeter and contents as a whole at the time of observation. Irregularity in the curve can almost invariably be traced to failure to fulfill one or both of these conditions. Thus a hump in the curve at *C* indicates that the bulb of the thermometer has been unduly heated by too close proximity to the source of heat. In all such cases the data should be rejected.

Necessary Data.—To illustrate the Regnault-Pfaundler method for computing the cooling correction which is adopted in this exercise, the following data will be employed:

Time	Temp. in Deg. Cent.				Time	Temp. C.°	
	Cal.	Subst.	Ex- posed Stem	Shield		Cal.	Shield
h. m. s.					h. m. s.		
2-00-00	15.91	99.6	19.8	17.4	M.P. 2-13-00	18.82	17.5
01	.94	99.6	19.9	.4	↓ 13-30	.82	.5
02	.96	99.7	20.0	.4	* 14-00	.81	.5
03	.97	99.8	20.0	.4	↓ 14-30	.80	.4
04	.99	.8	20.1	.5	15-00	.78	.5
05	.99	.8	20.0	.4	A.P. 16	.75	.5
P.P. 06	16.01	.8	20.0	.4	↓ 17	.74	.5
07	16.02	.8	20.1	.4	18	.71	.6
08	16.05	.8	20.1	.5	19	.69	.5
09	16.05	.8	20.1	.5	20	.68	.5
10-00	Substance introduced				21	.67	.5
10-30	17.41				22	.63	.5
11-00	18.41			17.4	23	.61	.6
M.P. 11-30	.71			.4	↓ 24	.60	.5
12-00	.82			.5	25	.58	.5
12-30	.83			.5	26	.56	.5
					27	.54	.5

In this experiment the time 2-10-00 marks the end of the Preliminary Period and the beginning of the Main Period. The temperature of the calorimeter was not read at this instant because the experimenters were too busily engaged in introducing the heated body into the calorimeter at the proper instant and in the correct manner. This temperature can, however, be readily determined by an inspection of the previous temperature readings or, better, from a suitable plot.

If a plot is made with the *corrected* readings of the calorimeter thermometer as ordinates and time as abscissas, a curve similar to Fig. 22 is obtained. The straight line *AB* is drawn to best represent the temperature change due to radiation before the introduction of the hot substance as shown by the data. The point where this line produced intersects the ordinate 2-10-00 is the temperature, t_0 (16.08°C.), of the calorimeter at the instant the substance was introduced. The point t' is one of the *earliest readings falling on this best representative line* and is arbitrarily chosen as the beginning of the new Preliminary Period. In a similar way the line *DE* is drawn to best represent the rate of fall of temperature of the mixture. Here t_n (18.81°C.) is a reading near the top of the curve which lies *in the straight line*. It is not necessarily the highest temperature reached. The point t'' corresponds to a reading near the end of the experiment *also*

falling in the straight line. These two points are chosen as the beginning and the end of the new After Period. The points t_0 and t_n then become, respectively, the beginning and end of the new Main Period.

The process of computing the radiation correction consists in determining the total gain or loss of temperature during the

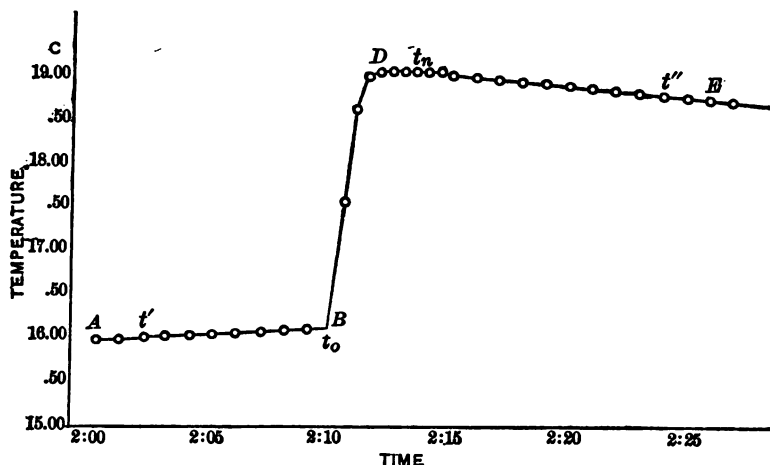


FIG. 22.

Main Period. This will be very nearly the algebraic sum of the gains of heat computed for each half-minute interval of this period. To compute these values it is necessary to know the rate of change of temperature of the Preliminary Period and of the After Period, together with the average temperatures of these periods.

*To Compute Radiation Correction by Graph.**—From Fig. 22 it is evident that the average temperature (θ') during the P.P. is $\frac{t' + t_0}{2} = \frac{15.95 + 16.08}{2} = 16.02^\circ \text{C.}$, and the average temperature (θ'') of the A.P. is $\frac{t_n + t''}{2} = \frac{18.81 + 18.60}{2} = 18.70^\circ \text{C.}$

During the P.P. the calorimeter was rising in temperature at the rate of $\frac{16.08 - 15.95}{2 \times 8} = 0.008^\circ \text{C. per half-minute.}$ During the

* *Note.*—The calorimeter temperatures used in the discussion are the *corrected* readings corresponding to the assumed observed data.

A.P. it was falling in temperature at the rate of $\frac{18.81 - 18.60}{21}$
 $= 0.010^\circ \text{ C. per half-minute.}$

Knowing the rate of gain or loss of temperature during the Preliminary Period and also during the After Period, at which times the only cause producing the change of temperature was radiation, the effect of radiation during the intervals of the Main Period may be computed. In obtaining this cooling correction, it is convenient to make use of what may be called a "radiation graph." (See Fig. 23.) On a piece of cross-section paper the average temperatures (θ' and θ''') during the preliminary and after periods are plotted as abscissas to a convenient scale. Then with a larger scale the corresponding average rates of temperature change (v' and v''') per half-minute interval during these periods are laid off as ordinates. These points should then

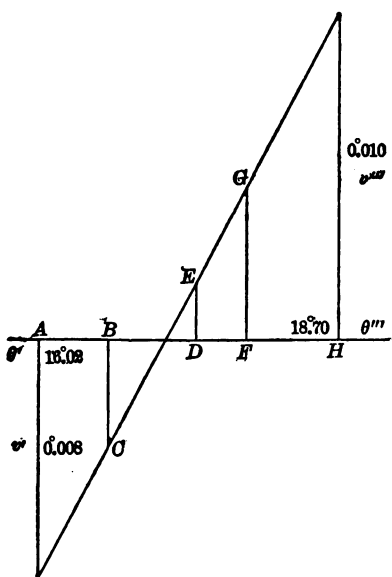


FIG. 23.

be connected by a straight line. From Newton's Law of Cooling, the rate of change of temperature per half-minute interval at any temperature F will be given by the ordinate FG . Gains in temperature are called *negative* and are measured downward, while losses in temperature are called *positive* and are measured upward. It has already been noted that at the time 2-10-00 when the heated body was introduced, the temperature of the calorimeter must have been 16.08° C. At the end of the first interval, 30 seconds later, the temperature was 17.40° C. We may, there-

fore, assume, without appreciable error for our purpose of correction, that during this interval the temperature was neither 16.08° nor 17.40° , but rather the mean of these two, i. e., 16.74° . Now referring to the graph, Fig. 23, we find that the ordinate corresponding to 16.74° is BC , and its length in terms of the scale

adopted is 0.003° . This means that during the interval 2-10-00 to 2-10-30 the calorimeter was rising in temperature to the very small amount of 0.003° C., on account of radiation of heat from the room to the calorimeter in addition to the rise due to the introduction of the heated body.

During the second interval of the Main Period the average temperature was 17.90° , and the ordinate DE of the curve at this point is about 0.005° . From this we see that during this interval of the Main Period, although the temperature of the calorimeter was rising, there was actually a loss of 0.005° on account of radiation. Similarly for the third interval the ordinate FG gives 0.009° as the loss of temperature. In this way the radiation loss for each of the remaining intervals of the Main Period is found, and by taking the algebraic sum of all these gains and losses we obtain 0.054° C., the total correction. *This correction, when added algebraically to 18.81° C., the last temperature of the Main Period, gives 18.86° C., the temperature which the calorimeter would have reached if there had been no radiation, i. e., if the introduction of the heated body had been the only cause producing changes of temperature.*

	Average Temp.	Radiation Corrections
$\frac{t_0 + t_1}{2} = \frac{16.08 + 17.40}{2} = 16.74^\circ \text{ C.}$		$- 0.003^\circ \text{ C.}$
$\frac{t_1 + t_2}{2} = \frac{17.40 + 18.40}{2} = 17.90$		$+ 0.005$
$\frac{t_2 + t_3}{2} = \frac{18.40 + 18.76}{2} = 18.58$		0.009
$\frac{t_3 + t_4}{2} = \frac{18.76 + 18.81}{2} = 18.78$		0.010
$\frac{t_4 + t_5}{2} = \frac{18.81 + 18.82}{2} = 18.82$		0.011
$\frac{t_5 + t_6}{2} = \frac{18.82 + 18.81}{2} = 18.81$		0.011
$\frac{t_6 + t_7}{2} = \frac{18.81 + 18.81}{2} = 18.81$		0.011
		$+ 0.054^\circ \text{ C.}$

To Compute Radiation Correction by Formula.—It is possible

to obtain the final result more rapidly and in general more accurately by combining all the operations which were performed above graphically into a single algebraic expression. A common form of this expression for *the total radiation or cooling correction is*

$$nv' + \frac{v''' - v'}{\theta''' - \theta'} \left(\frac{t_0 + t_n}{2} + t_1 + t_2 + t_3 + \dots + t_{n-1} - n\theta' \right)$$

where n = number of half-minute intervals in Main Period

v' = average change in temperature per half-minute interval in Preliminary Period, with its proper sign

v''' = average change in temperature per half-minute interval in After Period, with its proper sign

θ' = mean temperature in the Preliminary Period

θ''' = mean temperature in the After Period

t_0 = temperature of calorimeter at the moment the substance was introduced

$t_1, t_2, \dots, t_n$ = temperature readings taken every half-minute during the Main Period.

In the problem under discussion the numerical values of these quantities are:

$$n = 7 \qquad \theta' = 16.02^\circ \text{ C.}$$

$$v' = 0.008^\circ \text{ C.} \qquad \theta''' = 18.70^\circ \text{ C.}$$

$$v''' = 0.010^\circ \text{ C.} \qquad t_0 = 16.08^\circ \text{ C.}$$

$$t_1 = 17.40^\circ, t_2 = 18.40 \dots t_7 = 18.81^\circ \text{ C.}$$

Substituting these values in the above formula the *total radiation correction becomes*

$$\begin{aligned} 7(-0.008) + \frac{0.010 - (-0.008)}{18.70 - 16.02} & \left(\frac{16.08 + 18.81}{2} + 17.40 + 18.40 \right. \\ & \left. + 18.76 + 18.81 + 18.82 + 18.81 - 7 \times 16.02 \right) \\ & = -0.056 + \frac{0.018}{2.68} (17.45 + 111.00 - 112.14) \\ & = -0.056 + 0.109 \\ & = +0.053^\circ \text{ C.} \end{aligned}$$

The highest temperature of the calorimeter should have been, therefore, $18.81^{\circ}\text{C.} + .053^{\circ} = 18.86^{\circ}\text{C.}$ Throughout the above discussion it has been assumed that the temperature of the room (shield) remained constant or changed uniformly to a very slight extent.

Computations and Plots.—(1) If corrections are to be applied to the readings of the calorimeter thermometer, make a table on a fresh sheet of paper with the following data in four parallel columns: time, calorimeter-thermometer readings, corrections, calorimeter temperatures.

(2) Plot on the lower part of a sheet of cross-section paper the points showing the calorimeter temperatures during the experiment. Lay off time as abscissas and temperature (to a larger scale) as ordinates.

(3) Draw in the best representative straight lines for both Preliminary and After Periods, using a fine line on a strip of celluloid or a stretched thread to locate them.

(4) Determine points t' , t_0 , t_n , t''' , as the ones most nearly in the best representative straight lines. Connect t_0 and t_n by drawing short straight lines between all intermediate points.

(5) Next construct the radiation graph on the upper part of the sheet of cross-section paper used for the time-temperature plot. Choose a convenient scale of abscissas and the scale of ordinates sufficiently large. From this graph compute the radiation correction as described above.

(6) Obtain a check value for this radiation correction, using the formula given in the *General Discussion*.

(7) Finally, with the average of these two corrections, compute the corrected final temperature of the calorimeter and contents, and then the specific heat of the substance. As the thermometer which recorded the temperature of the hot substance had a number of degrees of stem exposed, a correction must be applied to this thermometer for exposed stem as well as for standardization.

Problem.—Assuming the simplest condition where v' and v''' are both on the same side of the X axis of the graph (see Fig. 23), deduce the formula used in this exercise for computing the total radiation correction.

EXERCISE 27

MECHANICAL EQUIVALENT OF HEAT

Object.—The determination of the mechanical equivalent of heat by a mechanical method.

General Discussion.—The mechanical equivalent of heat expresses the ratio between any mechanical unit of energy and a heat unit, for example the ratio between the foot-pound and the B.t.u., the erg and the gram-calorie. Symbolically $J = \frac{W}{H}$, where W is expressed in units of work and H in heat units. The value of J may be determined experimentally by converting mechanical or electrical energy into heat.

In the Callendar apparatus which is used for this experiment, heat is developed by friction between a cylindrical metal drum, which is rotated, and straps of tape wound around this drum. A weight of 4 or 5 kilograms is attached to one end of the system of tapes, and the drum is rotated at such a rate that the frictional force exerted on the tapes by the rotating drum will just support this weight. Several small (50 gram) weights attached to the other end of the tapes keep them tightly wrapped around the drum.

To adjust automatically the tension of the tapes, part of the force exerted by the smaller weights is borne by a spring balance attached to the framework of the apparatus. When the friction between the tapes and the drum becomes too great, the large weight is lifted higher and the small weights descend on their side. This puts more of their weight upon the spring balance, thus lessening the tension on the tapes and decreasing the frictional force.

The amount of mechanical energy expended in frictional heating of the drum and its contents is equal to the force opposing the motion of the drum multiplied by the distance through which a point on the surface of the drum moves during this experiment. The force which opposes the motion of the drum is equal to the weight of the large mass decreased by the weight of the 50 gram weights used on the other side and increased by the reading of

the spring balance. The distance through which this force is overcome can be computed from the diameter of the drum and the number of revolutions it makes, as indicated by a revolution counter attached to the apparatus.

The drum is partly filled with water, and the heat developed by the friction of the tapes on the drum raises the temperature of the water and drum. This change in temperature is measured by a thermometer inserted in the water. Using the heat capacities of the drum and water, the amount of heat energy which has been supplied to them to produce the observed rise in temperature is readily calculated; and from this and the amount of mechanical energy expended in producing this heat, the value of the mechanical equivalent may be found.

Apparatus.—Callendar apparatus, thermometer and flask.

Manipulation.—(1) Make sure that the drum contains no water. The outlet is near the rim of the drum and is closed by a small screw.

(2) Adjust the weights (using at least 4 kilograms as the larger weight) so that on turning the drum the larger weight will be raised until the cross-bar to which it is attached remains practically stationary at about $\frac{1}{2}$ to 1 inch above the floor of the machine.

(3) Fill the drum with 250 grams of water, the temperature having been adjusted as discussed below, by using a 250 c.c. flask.

As it is impossible to prevent radiation errors, some method should be employed for their elimination. The production of heat is fairly uniform, and may be discontinued at any time, hence the *Rumford method* of correction may, in general, be readily applied. This consists in starting with the water at a temperature 2° to 3° *below* the temperature of the room and continuing the experiment until a temperature just as much *above* room temperature is obtained. Under these conditions as much heat is *gained from* the surroundings during the first half of the experiment, while the temperature of the drum and water is *below* that of the room, as is *lost to* the surroundings during the last half, while the temperature is *above* that of the room.

In case the humidity in the room is high, it may be found that the cooling of the drum by putting in it water cooler than the sur-

rounding air will cause the deposition of some water on its surface. This will cause the tapes to stick more closely to the drum, so that the rate of turning necessary to support the heavy weight will be very different after the water has been put in the drum. Under such conditions, as it is obviously impossible to use the Rumford method, the drum should be filled with water at *approximately room temperature* and the radiation error corrected as follows: Remove the weights and turning the drum read the temperature of the water in the drum every half-minute for six minutes to obtain the rate (in degrees per minute) at which the temperature rises or falls, owing to gain or loss of heat to the surrounding bodies. This rate should be very small, if the temperature of the water has been properly adjusted. Then replace the weights and proceed with the experiment, turning the drum until a temperature rise of 4° or 5° has been obtained. Then, immediately after recording the necessary data for the experiment proper, remove the weights and, *revolving the drum*, take another series of temperature readings extending over six minutes, and from these obtain the rate at which the temperature falls. The mean of this and the initial rate of fall, multiplied by the duration of the experiment proper, will give the loss in *temperature* due to radiation. This correction, then, added to the observed rise in temperature gives the amount the drum and water would have been heated by the mechanical energy expended if they had been isolated from all surrounding objects.

Precautions.—Record all weights used and the reading of the counter attached to the instrument before starting the experiment. All temperatures in this exercise should be read to 0.02° C. Be careful that the large weight is continually supported by the frictional force, and that neither cross-bar strikes the iron floor of the apparatus. Record the average reading of the spring balance during the turning.

Make three independent determinations of the mechanical equivalent.

Computation.—(1) The force opposing the motion and the distance through which this force is overcome can be obtained as described above. Expressing these quantities in the proper units compute the mechanical energy supplied in ergs.

The water equivalent of the drum can be computed from its mass, which is stamped on it, and its specific heat, which is 0.088

calorie per gram. With this value, the mass of water used and the corrected rise in temperature, compute the amount of heat energy developed in gram-calories. Then compute the value of J in ergs per gram calorie.

(2) Using the mean of the values of J in ergs per gram-calorie, compute the mechanical equivalent expressed (1) in kilogram-meters per (large) Calorie (1000 gram-calories); (2) in foot-pounds per B.t.u.

EXERCISE 28

CONSTRUCTION OF VOLTMETER AND AMMETER

Object.—(1) To become acquainted with the construction, theory and use of a direct-current voltmeter, to construct such an instrument from a suspended coil galvanometer, and to calibrate it by comparison against a Weston voltmeter taken as a standard. (2) To determine with the galvanometer-voltmeter and a resistance box the internal resistance of a cell. (3) To convert the suspended coil galvanometer into a direct-current ammeter and to calibrate it. (4) To measure an unknown resistance by means of the calibrated galvanometer-voltmeter and galvanometer-ammeter.

General Discussion.—The use of series resistances (extension coils) with millivoltmeters and of shunts with milliammeters to vary the range of such instruments has been discussed in the theoretical course on electricity. The student should review these topics and also the relation of electromotive force, internal resistance and terminal voltage of a cell before attempting to perform this experiment.

For a more extended discussion of the subject read Brooks and Poyser, Chapter XXXIII, on Measuring Instruments, and Pender's American Handbook for Electrical Engineers, articles on Ammeters and Voltmeters.

Wall type d'Arsonval galvanometers require, to cause one centimeter deflection on a scale 50 cm. distant, voltages across the galvanometer terminals of the order of 3×10^{-4} to 3×10^{-6} volt, while their resistance is such that the corresponding currents through the instruments for this deflection are on the order of 3×10^{-6} to 3×10^{-9} ampere. This high sensitivity makes it possible to convert them into very high resistance voltmeters or into very low resistance ammeters, which is often advantageous. It also necessitates great care against impressing directly on the galvanometer terminals voltages such as those of a primary or a secondary cell. For a discussion of sensitive galvanometer construction and characteristics, see Leeds and Northrup catalogue No. 20.

Apparatus.—A d'Arsonval galvanometer; a variable high resistance; a brass rod with movable contacts to be used as an ammeter shunt; a resistance box variable by steps of one ohm from 1 to 10 ohms with the coils especially designed to dissipate as much as one watt without injury due to overheating; a Weston voltmeter (range 0–3 volts) of resistance about 300 ohms; an unknown wire resistance stretched over a meter scale; a storage cell; necessary switches and connecting wires.

Manipulation and Computations.—PART I.—(1) Conversion of a d'Arsonval galvanometer into a voltmeter of range 0 – 2.5 volts.

Connect the galvanometer and adjustable high resistance in series through a knife switch to the terminals of the storage cell. Connect the Weston voltmeter through a switch to the same terminals. Assuming that the Weston voltmeter reads the correct voltage between its terminals, adjust the high resistance until the galvanometer scale registers 10 cm. for each volt of the Weston reading when both instruments are connected to the cell at the same time. This method assumes that the deflections are directly proportional to the terminal voltage throughout the range, a very fair assumption provided the galvanometer scale is circular. *Record the resistance required so that it may be inserted whenever the galvanometer is subsequently used as a voltmeter.*

Note.—If the galvanometer is provided with a fixed external shunt, this shunt must not be removed throughout the exercise.

(2) Comparison of the galvanometer-voltmeter with the Weston instrument.

(a) Take readings on the Weston voltmeter before and after opening the galvanometer switch. (b) Take readings on the galvanometer before and after opening the Weston switch.

How do you explain any differences that may be observed?

Which of the four readings corresponds most nearly to the e.m.f. of the cell?

PART II.—Determination of the internal resistance of the cell.

A storage cell has a very low internal resistance. The cell used in this exercise has, therefore, a permanently connected series resistance to prevent injury to the cell or other apparatus in case of a short circuit. This resistance plus the true internal resistance is what will be measured as internal resistance in this exercise.

Draw a diagram showing a cell of unknown resistance R_1 supplying current through a known resistance R_2 . Devise a general method for computing R_1 in terms of R_2 , the voltage drop V_2 across the terminals of R_2 , and the e.m.f. E of the cell, i. e., the terminal voltage on open circuit. Make connections according to the diagram and measure with the galvanometer-voltmeter the e.m.f. of the cell, also the voltage drops V_2 corresponding to $R_2 = 4, 6, 8$ and 10 ohms. Calculate R_1 from each set of readings and compute the A.D. of the mean. Compute the current when $R_2 = 0$, and estimate the precision of this result.

PART III.—To adjust and calibrate the galvanometer as an ammeter.

The shunt for this purpose consists of a brass rod having four contact points, two fixed contacts at the end for the current connections and two contacts adjustable along the rod for the galvanometer terminals. Connect this shunt, with the galvanometer attached, across the terminals of the cell whose internal resistance has just been computed. If the resistance of the brass rod is less than 0.005 ohm, will its introduction into the circuit have an appreciable effect on the value of the current as computed in Part II?

Adjust the movable contacts until the galvanometer shows a deflection of 10 cm. per ampere of the calculated current. A knife-switch should be in the circuit to cut out the cell when not in use.

PART IV.—Measurement of an unknown resistance.

Connect through a switch a wire (stretched over a meter stick) in series with the cell and the galvanometer-ammeter. Read and record the current.

Disconnect the galvanometer connections from the shunt and again adjust the galvanometer as a voltmeter. With the galvanometer-voltmeter, record for the constant current just measured the potential differences (P.D.'s) between points on this wire $20, 40, 60, 80$ and 100 cms. apart. Plot a curve between lengths of wire and corresponding potential differences. Does the curve pass through the origin? Should it?

From the measurements on the wire compute the resistance of 100 centimeters of the wire and estimate the precision of this result.

Discuss the sources of error in the voltmeter-ammeter method of measuring resistance.

The report should contain neat diagrams of the connections in every case.

Problems.—1. (a) Given a d'Arsonval galvanometer of resistance 100 ohms which requires 3×10^{-6} volt between its terminals to produce a deflection of 6 cm. Compute the necessary resistance to convert it into a voltmeter giving 10 cm. deflection per volt. Show diagram of connections.

(b) Compute necessary resistance to convert this galvanometer into an ammeter with 10 cm. deflection for 1 ampere in main circuit and give diagram of connections.

2 A storage cell with no appreciable internal resistance has an e.m.f. = 2.2 volts. This cell is connected to the terminals of two resistances *A* and *B* connected in series, whose resistances are 100 and 1000 ohms respectively. (1) What are the P. D.'s between the terminals of *A* and *B*? (2) What will a 100 ohm voltmeter read when connected across *A* and when connected across *B*? (3) What will a 10,000-ohm voltmeter read when connected across *A* and then across *B*?

1.425

2.2

.561

Answer

EXERCISE 29

THE WHEATSTONE BRIDGE.—SLIDE WIRE AND DECADE FORMS

Object.—To become acquainted with the elementary principles of the Wheatstone bridge as exemplified in the slide-wire and the box forms. To verify the relation of resistances in series and in parallel.

General Discussion.—Before performing this exercise read Brooks and Poyser, pages 261–275, also Duff, pages 390–402 (fourth edition). For a more detailed discussion of methods of measuring resistance, see *Methods of Measuring Electrical Resistance* by E. F. Northrup, or *Electrical Measurements* by F. A. Laws.

The international ohm is defined as the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of a length 106.300 cm. The international ohm is based upon but is not legally defined as equal to 10^9 c.g.s. electromagnetic units of resistance. Copies of this international ohm or multiples of it are made of suitable materials, and resistance measurements nearly always consist in making comparisons with such copies. Manganin (84 per cent. copper, 12 per cent. manganese and 4 per cent. nickel) is a very suitable alloy for resistance coils. Its resistivity at 20°C . is about 45×10^{-6} ohm per centimeter cube, which is nearly 25 times that for copper. Its mean temperature coefficient of resistance between 15°C . and 20°C . is about 0.000005 compared with about 0.004 for copper, and its thermo-electromotive force when opposed to copper is only 0.000002 volt per degree C.

Resistance measurements may be divided into three general classes, namely, those (1) of low resistance, less than 1 ohm; (2) of high resistance, a megohm (10^6 ohms) or greater; (3) of medium resistance.

The determination of low resistance, such as the resistance of an ammeter shunt or of a bus-bar, requires special care and apparatus, the method used being one that eliminates contact and plug resistances. The Kelvin double-bridge is designed for

this purpose. There are also special methods adapted to the measurement of high resistances, and others for the comparison of two resistances which are very nearly equal. For methods of measuring the resistance of electrolytes, see Northrup, pages 238-248.

If the slide-wire bridge is to be used with precision, the wire itself must be carefully calibrated throughout its length and the effect of end resistances, where the wire is connected to the heavy straps, must be known and taken into account. (See Laws, *Electrical Measurements*, pages 173-174.) The thermoelectromotive forces may be eliminated by reversing the cell terminals. When all possible precautions are taken, the slide-wire bridge has high precision possibilities. The Carey Foster bridge, a modification of the simple slide-wire form, is primarily designed for the comparison of nearly equal resistances and, therefore, lends itself to the measurement of temperature-resistance coefficients and to the verification of standard resistances.

In this exercise the slide wire is to be used only as a method of illustrating the fundamental principles of the Wheatstone bridge, and not as an instrument of precision.

The ordinary box bridge is suited to the determination of resistances of medium value where the desired accuracy is not excessive, say not better than 0.2 per cent. In a good commercial form of box type of Wheatstone bridge the makers will adjust the rheostat coils to an accuracy of 0.1 per cent. and the ratio coils to an accuracy of 0.05 per cent. The bridge method is a null method and as such requires a galvanometer with a sensitivity in keeping with the desired precision. The sensitivity of the d'Arsonval galvanometer is usually greater than that required by the accuracy of the results.

In making a standard resistance box which may be conveniently varied in small steps over a wide range, it is desirable to have as few standardized coils as possible with as little plug resistance as possible. Many forms have been designed with this end in view. One of the best of these designs is shown in Fig. 25, where only four coils and one plug are used for obtaining any whole number of ohms from 0 to 9 ohms. (The student should satisfy himself, by tracing out connections, that the plug inserted in any gap gives the resistance indicated on the corresponding bar.) Similar groups could be made with the coils having 0.1 or 10 times

or 100 times the values shown in the figure, and these groups or decades, as they are called, could be joined in series. This general type of connection also lends itself to a dial arrangement with sliding spring contacts, which seem to be even better than plugs.

One of the Bureau of Standards circulars assumes probable variation for *contacts kept in good condition* to be:

Binding post.....	0.00002 ohm
Plug.....	0.0001 “
Switch.....	0.0002 “

In practice these contact resistances are apt to be larger. It should also be remembered that each foot of the copper lead wires may have on the order of 0.003 or 0.004 ohm resistance.

Apparatus.—Some unknown resistances; box of known resistances; table galvanometer; double-contact key; slide-wire bridge; a box bridge (decade form); universal shunt; a wall type d'Arsonval galvanometer; a dry cell.

Manipulation and Computations.—PART I.—Connect the slide-wire bridge according to the customary diagram, using the double contact key. This key is made to close the battery circuit first and then the galvanometer circuit, and works in the reverse order on opening the circuit. If two separate keys are used, the same procedure must be followed. *Why?* The galvanometer should be quick acting with a short period, and the circuit should be kept closed only long enough to detect the direction in which the needle moves. A continuous current will generate heat, changing the value of the resistances, and may injure the apparatus.

Place any known resistance in one arm of the bridge and measure the resistance of the unknown coils. Then adjust the known resistance so that the bridge will balance as near the center of the slide-wire as possible. *Why?* Maintaining this value of the known resistance constant, make three or four independent settings of the sliding contact.

PART II.—Repeat Part I with the cell and galvanometer interchanged on the bridge. How do you explain the results?

PART III.—Measure the same resistance with the box bridge, making connections as indicated on the bridge. Use the wall galvanometer with the universal shunt for this and the remaining parts of the exercise.

The terminals of the galvanometer are to be connected to the plugs marked *G* on the shunt-box. The terminals marked *Ga* on the bridge are to be connected to the terminals marked *L* or sometimes *Ba* on the shunt-box. The working of the shunt

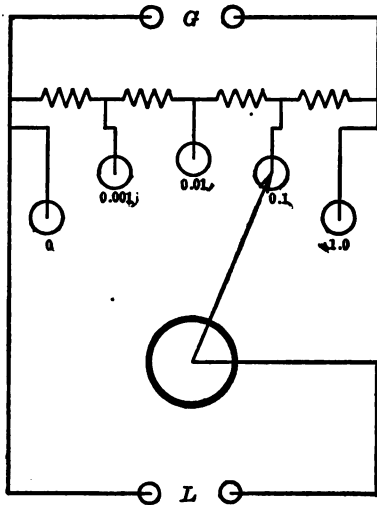


FIG. 24. Universal Shunt.

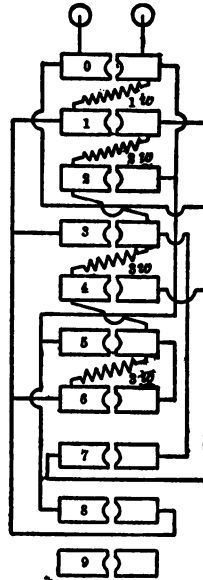


FIG. 25. Northrup's decade arrangement for resistance coils.

box will be evident from Fig. 24. Until a balance on the bridge is approximated, the shunt should be set at 0.001 or 0.01, and finally a careful balance made with the shunt on 1.0.

Short circuit the unknown resistance and determine the resistance of the leads and contacts.

PART IV.—Carefully measure another given unknown with the decade box and correct for resistance of leads.

PART V.—Measure the resistance of the two unknown coils when connected (1) in series, (2) in parallel. Correct each of these for the resistance of the leads, and compute the percentage difference between the measured series and parallel values and the values as computed from the corrected measurements on the separate coils in Part III and Part IV.

The student should be prepared to discuss the theory of the Ayrton shunt.

Problem.—Given the two equations,

$$R = a + b \quad \text{and} \quad R' = \frac{ab}{a + b}$$

Let $a = 3.00 \text{ ohms} \pm 0.01\omega$, $b = 5.00 \text{ ohms} \pm 0.01\omega$. Compute the precision of R and R' in ohms and in per cent.

EXERCISE 30

THE POTENTIOMETER

Object.—A study of the construction and principle of the potentiometer, and of its application to the measurement of electromotive force.

General Discussion.—Consider a cell E_1 , a current detector G , a resistance R , a key K_2 joined together and connected across a portion A of a resistance CD in which flows a current due to a source of constant e.m.f. E . Call the resistance of A in this

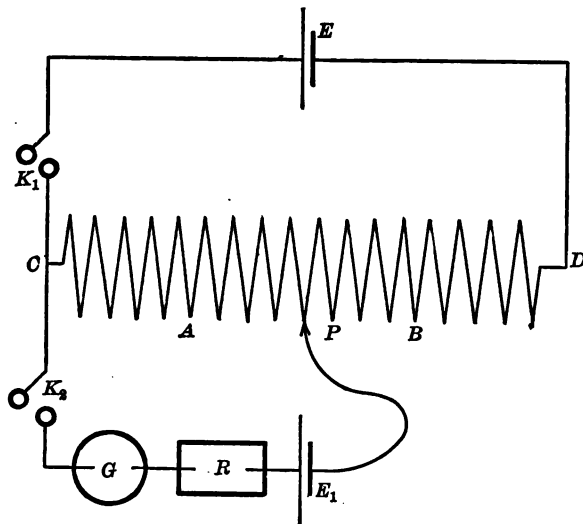


FIG. 26.

case R_1 . If E_1 be in opposition to E and the IR drop along A be equal to E_1 , no current will flow in CGP on closing K_2 . If E_1 be replaced by a cell E_2 , a new position of P , say P' , (hence a new value of $A = R_2$) can be found such that there will be a balance, i. e., no current will flow through CGP' . If the current I in CD is constant, it follows that $E_1 : E_2 = R_1 : R_2$. If E_1 is known, then

$$E_2 = E_1 \frac{R_2}{R_1}. \quad (1)$$

Note.—In the potentiometer E_1 and E_2 must always be less than E , and the cell E must be free from polarization to maintain the current I constant.

Slide-Wire Form of Potentiometer.—If CD be a wire of constant cross-section, equation (1) becomes $E_2 = E_1 \frac{l_2}{l_1}$, since R_1 and R_2 are proportional respectively to the lengths l_1 and l_2 of CP .

The slide-wire form of potentiometer may be so arranged as to become direct reading, i. e., so that the value of the unknown electromotive force E_2 is the reading l_2 on the scale with the decimal point in the proper place. Read S. P. Thompson's *Elementary Lessons in Electricity and Magnetism*, part b of section 448, page 426 (1915 edition).

Standard Resistance Form of Potentiometer.—For the parts A and B of CD two standard resistance boxes, preferably identical, are used. For theory of method see Brooks and Poyser, page 306. By a method exactly similar to that for the slide-wire form, the standard resistance form of potentiometer may be made direct-reading.

"Electromotive forces are measured in terms of Weston cells. The e.m.f. of the mean Weston normal cell is fixed by the International Committee on Electrical Units and Standards as 1.0183 international volts at 20° C. Since January 1, 1911, this value has been used in all official measurements at the Bureau of Standards. The mean Weston normal cell is determined by comparison of cells set up in the four different national laboratories, taking the mean value as the value of the mean Weston cell. The differences between the reference cells of the different national laboratories are not more than two or three hundred thousandths of a volt."*

The Weston cells have a long life and a constant e.m.f. with a small temperature coefficient. The e.m.f. at any temperature t is

$$E_t = E_{20} - 0.0000406 (t - 20) - 0.00000095 (t - 20)^2 + 0.000000001 (t - 20)^3$$

The potentiometer should be used whenever high precision is desired in the measurement of electromotive forces, and particularly for the measurement of low voltages such as thermo-electromotive forces. With a sensitive galvanometer values as small

* Circular of the Bureau of Standards, No. 6.

as 0.001 millivolt can be measured. For a description of one type of precision potentiometer, see Leeds and Northrup catalogue No. 70.

Another important use of the potentiometer is for the calibration of direct-current ammeters and voltmeters.

Apparatus.—A precision potentiometer; a ten-meter slide-wire potentiometer; a table galvanometer; protective resistance for the galvanometer; a secondary standard cell; cells of unknown e.m.f.; standard dial resistance boxes; storage cell; switches and lead wires.

Manipulation and Computation.—PART I.—Connect the ten-meter slide-wire as shown in Fig. 26 and measure the e.m.f.'s of three unknown cells. The wires of the slide-wire potentiometer have enough resistance so that the storage-battery circuit may be kept closed throughout the exercise. This allows the wires to reach a constant temperature and resistance. Make four independent settings for each cell and note the sensitivity of the apparatus, i. e., the largest variation in setting that can be made without noticeably affecting the galvanometer. If the readings show a progressive change in one direction, consult the instructor.

PART II.—Substitute for the slide-wire the dial boxes in series. The unknown and standard cells are to be set in turn across the terminals of *one* of the boxes with their e.m.f. in opposition to the P.D. of the storage-battery circuit. Let this box be called *A* and the other *B*. To make the method *direct-reading*, connect the standard cell with the protective resistance, galvanometer and key across *A*, and set *A* at an even multiple of the standard cell voltage, say 1000 or 10,000 times this voltage. Then adjust *B* until the galvanometer shows no deflection. The fall in potential will then be 1 or 0.1 millivolt per ohm in the storage battery circuit. For example, if the standard cell has an e.m.f. = 1.0183 volts, set *A* at 10,183 ohms. If the storage-battery voltage is about 2.2 volts, then the resistance in *B* should be about 12,000 ohms, the total resistance *A* + *B* being approximately 22,000 ohms. Whatever the *total resistance* may be, it *must be kept constant* when the other cells are substituted for the standard.

This procedure assumes a constant e.m.f. for the storage battery, which constancy should be checked up after measuring the unknowns by again testing with the standard cell.

Arrange the data in four columns, one for the resistance A , one for B , one for $A + B$, and one for the e.m.f.'s of the unknown cells. Make four settings on each cell.

Determine the limits of precision imposed by the sensitivity of the galvanometer.

Questions.—1. In Fig. 26 is it essential that K_1 be a switch of unvarying resistance? How about switch K_2 ?

2. Why not use a universal shunt with the galvanometer instead of the resistance R ?

3. What is the largest voltage that can be measured with the apparatus as you used it in Part II? Can you suggest a method of extending the range of this potentiometer to include the measurement of e.m.f.'s up to 120 volts?

4. Hand in solutions of problem 47 and problem 48 on precision.

EXERCISE 31

COMPARISON OF CAPACITIES BY BALLISTIC GALVANOMETER

Object.—(1) The comparison of capacities and measurement of quantity of electricity by means of the ballistic galvanometer. (2) The comparison of the electromotive force of cells by the use of a condenser and a ballistic galvanometer.

General Discussion.—When a quantity of electricity is *suddenly* discharged through a d'Arsonval galvanometer, a certain amount of angular momentum is communicated to the moving coil. The student should be prepared to demonstrate that this angular momentum is proportional to the quantity of electricity, i. e.,

$$I\omega = (\text{Const}) \times Q$$

The following discussion shows that the maximum deflection or first "throw" of the moving coil is proportional to the initial angular momentum, and therefore to the quantity Q of electricity discharged through the coil. Neglecting the damping, which is usually small, the initial kinetic energy $\frac{1}{2}I\omega^2$ of the coil is equal to the work done in producing the maximum angular deflection or first "throw" θ_1 of the coil. The torque opposing the angular motion of the coil is due to torsion in the suspension. This torque is proportional to the angle of twist and equals $K\theta$ where K is the constant of torsion. Hence the total work done in twisting the coil through an angle θ_1 is

$$\int_0^{\theta_1} K\theta d\theta = \frac{1}{2}K\theta_1^2$$

Therefore

$$\frac{1}{2}I\omega^2 = \frac{1}{2}K\theta_1^2$$

or

$$\omega = (\text{Const}) \times (\theta_1)$$

Consequently $\theta_1 \propto Q$, or $Q = X\theta_1$. This relation holds even if damping exists, but the value of the constant X will change with change in the amount of damping.

In practice a special form of galvanometer, called a "ballistic" galvanometer, is used to measure quantity of electricity. This

type has a moving coil of large moment of inertia and hence a long time of vibration, so that the entire charge of the condenser passes through the coil before any appreciable angular deflection occurs. These galvanometers generally, although not necessarily, have little damping.

For a more extended discussion of ballistic galvanometers, see Brooks and Poyser, Chapter XXIV with the Appendix to the chapter, also the Leeds and Northrup Catalogue No. 20.

Apparatus.—A ballistic galvanometer; a standard mica condenser; several condensers of unknown capacity; a three-point key; several cells; a high resistance.

Manipulation and Computation.—(1) Compare with the standard condenser the capacity of each of the unknown condensers. For connections, see Fig. 27. The condensers should be charged

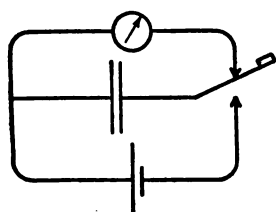


FIG. 27.

for ten seconds each time, as the deflection may be found to vary with the length of the charging period. Each deflection should be determined several times and the mean used in the calculations.

If a very sensitive galvanometer is used, it may be necessary to shunt it.

However, if a shunt is used, it must be employed for all deflections that are to be compared. For "Use of Shunts," see note.

(2) Measure the capacity of combinations of condensers when connected (1) in series, (2) in parallel, as directed by the instructor. Compare these experimental results with the values computed from the capacities of the separate condensers.

(3) Measure the electromotive force of a good cell with a high resistance laboratory voltmeter. With this cell and the standard condenser determine the *sensitivity* of the galvanometer in *micro-coulombs per millimeter deflection*.

Then, using the standard condenser, determine the electromotive forces of the other cells.

What precision can be obtained with this method (1) for comparing capacities, (2) for comparing electromotive forces?

(4) Choose a combination of condensers which will give a large deflection. Observe the deflections produced by discharg-

ing this condenser combination after charging it with one of the cells for periods of $\frac{1}{2}$, 1, 2, 5, 10, 15, 20, and 30 seconds, respectively. Make similar measurements when the connection with the cell is made through a resistance of several hundred thousand ohms. In each case plot a curve showing the relation between the time of charge and the observed deflection, using the same scale and the same axes for both curves.

The dielectric in the condenser takes on what is known as an absorption charge and thus plays a large part in determining the shape of these curves, that is, in determining the time required to charge a condenser to a given potential difference. The dielectric is, therefore, an important factor when condensers are used in high frequency alternating circuits.

Note.—Use of Shunts with Ballistic Galvanometer. It has been shown that when different quantities of electricity are discharged through the ballistic galvanometer the deflections observed are proportional to the quantities, provided the damping of the instrument has not meanwhile been altered.

The damping is largely due to the current produced by the motion of the coil in the magnetic field. If the instrument is used without a shunt to measure the discharge of a condenser, the circuit is not closed, and hence no appreciable induced current flows. If a shunt is used, any alteration of the *resistance connected across the galvanometer terminals* will change the constant of proportionality between deflection and quantity. The Ayrton (Universal) shunt may be used to vary the sensitiveness of the galvanometer, since in this shunt the resistance in the galvanometer circuit is fixed. Other forms of variable shunt should be avoided. It is to be noted that the shunt set on point 1 is not the same as no shunt. See diagram in Exercise 29.

$$Q = \tau V$$

EXERCISE 32

THERMO-COUPLE AND SENSITIVITY OF A GALVANOMETER

Object.—(1) To study the variation of the e.m.f. of a thermo-couple with the difference in temperature between the hot and cold junctions. (2) To measure the sensitivity of a galvanometer.

General Discussion.—In preparation for this experiment the student should review Brooks and Poyser, Chapter XXVIII.

The sensitivity of a galvanometer may be expressed in various terms. One method is to state the voltage which must be applied to its terminals to produce unit deflection, called "voltage sensitivity." "Unit deflection" is usually taken to mean a deflection of 1 mm. as read on a scale situated one meter from the mirror. In the case of galvanometers like that used in this experiment, where the scale is fixed at a shorter distance from the mirror, we may express the sensitivity as the voltage required to produce unit deflection *on this scale*. The "current sensitivity," that is, the current which must flow through the galvanometer in order to give 1 mm. deflection, is readily found if the voltage sensitivity and the resistance of the galvanometer are known. The "ohm (or megohm) sensitivity" is the resistance (in ohms or in megohms) through which one volt will cause a deflection of 1 mm.

Apparatus.—A copper-iron thermo-couple arranged so that one junction may be kept at a constant temperature while the other is heated; thermometers; a sensitive galvanometer; a variable resistance box to be used in series with the galvanometer; a resistance box containing high and low resistances in series; a dry cell; a voltmeter.

Manipulation.—PART I.—Heat one junction of the thermo-couple to about 285° C. Insert the variable resistance box in series with the galvanometer and thermocouple, and adjust its resistance so that for this temperature the deflection is about 20 cm. Allow the junction to cool. Take readings on the galvanometer at intervals of about 20°. Also record occasionally the temperature of the cool junction. Continue these readings

until the temperature difference between hot and cold junctions has fallen to about 25°C . Break the galvanometer circuit after each reading by means of the knife-switch and reset on zero of scale. If bothered by the continued oscillation of the galvanometer, short-circuit the instrument. How do you account for this damping action?

PART II.—The second part of this experiment consists in calibrating the galvanometer, with the same resistance in series as used with the thermocouple, so as to obtain the e.m.f. of the thermocouple in microvolts. To do this, remove the leads from the thermocouple and impress on the galvanometer circuit (galvanometer with resistance) known low voltages. These voltages are to be obtained by means of a cell of known voltage and a box provided with traveling plugs, containing known high and low resistances in series. Make a diagram of the connections and show to an instructor before proceeding with this part of the experiment. Vary the voltages applied to the galvanometer circuit in such a way as to get deflections over the same range as those produced by the thermocouple. Record the IR drops and the corresponding deflections.

PART III.—By means of the variable known resistance in series with the galvanometer, change the current through the instrument, keeping the voltage applied to the galvanometer circuit constant. On the assumption that the deflection is proportional to the current, calculate the resistance of the galvanometer. Then calculate the current flowing through the galvanometer. An easy method of determining the resistance of a galvanometer, where the voltages are low enough to allow it, is to obtain the deflection with no resistance in series, and then to add resistance until the deflection is cut to half value. *Explain.*

Computation and Plots.—Plot the data from Part I to show the relation between deflections and temperature differences. Plot deflections as ordinates. Then with the same axes plot the data from Part II to show relation between deflections and microvolts (millionths of a volt). From these plots determine the microvolts generated by the thermocouple for temperature differences of 50° , 100° , 150° , 200°C .

From Part III compute (1) the resistance of the galvanometer, (2) the microamperes per millimeter deflection, (3) the megohm sensitivity in megohms per millimeter deflection, (4) the microvolts per millimeter at the galvanometer terminals.

EXERCISE 34

ELEMENTARY PRINCIPLES OF THE GENERATOR

Object.—To investigate the e.m.f. induced in a conductor moving in a magnetic field.

In preparation for this exercise the student should read Brooks and Poyser, pages 348–356; also Duff, pages 437–448, 450–457, and be prepared to show that the

$$\text{induced e.m.f.} = \frac{1}{10^8} \cdot \frac{d\phi}{dt} = \frac{lvH \sin \theta}{10^8} \text{ volts,}$$

where l = length of conductor, H = magnetic field strength, v = linear speed, θ the angle between the velocity of the moving conductor and the direction of the magnetic flux.

Apparatus.—The apparatus consists of a two-pole generator. The armature of the generator consists of brass rods insulated from the core, unconnected on one end and joined to a common axis on the other end. One brush is in contact with this common axis, and the other brush is arranged to make contact at any desired angle in the field with the free end of each conductor as the armature rotates. From the brushes wires are led through a shunt box to a D'Arsonval galvanometer. The connections will be evident upon inspection of the apparatus. The field of the dynamo is fed from the 110 volt line. A bank of lamps in series with the field coils is used to vary the strength of the field current.

The generator is motor-driven, the motor being supplied with a speedometer and a variable resistance for regulating its speed. With this apparatus the following parts of the experiment should be carried out, remembering that the galvanometer deflection is proportional to the induced e.m.f.

Manipulation and Plots.—PART I.—Use the hard rubber core armature and only the *residual magnetism* of the field. Set the position of the brush at a point such that with a maximum speed the deflection will not be too great for the galvanometer. Leaving the brush fixed in this position, find the variation of the e.m.f. with the speed of rotation. Plot a curve between these two variables.

PART II.—Excite the field of the generator with a current not greater than 0.5 ampere. Leaving the field strength and speed constant, record the variation in the induced e.m.f. with the angle of cutting by setting the brush at intervals of about 15° all around the circle. Plot a curve between the angle expressed in degrees and the induced e.m.f.

Note.—Before beginning this set of readings, make sure that the shunt box is so adjusted as to allow the largest deflection to be read.

PART III.—Leaving the speed and position of brushes constant, vary the field strength by adding lamps one by one. Plot the curve showing the relation between the induced e.m.f. and the field current. Does it pass through the origin? Why?

If time permits, try Part II with one of the iron core armatures.

Conclusions.—In the report explain the meaning of the three different curves, and discuss them in terms of the relation

$$e = \frac{wH \sin \theta}{10^8}$$

Draw carefully a straight line diagram of the electrical connections. For symbols see the chart posted in the laboratory.

EXERCISE 35

CALIBRATION OF BALLISTIC GALVANOMETER BY STANDARD SOLENOID AND MEASUREMENT OF MAGNETIC FLUXES

Object.—The object of this exercise is to calibrate a ballistic galvanometer when in series with a test or flip coil for the measurement of magnetic field intensity and of magnetic flux, and to use the calibrated galvanometer in a study of magnetic hysteresis.

General Discussion.—Read the General Discussion of Exercise 31. Other references: Brooks and Poyser, pp. 368–378.

The number of linkages of a circuit may be defined as the sum of all the lines of magnetic induction or of magnetic lines of force linked with the circuit. Whenever the number of linkages, λ , is altered, an instantaneous e.m.f. is set up in the circuit whose magnitude is proportional to the *time rate of change* of the number of linkages, i. e.,

$$e \propto \frac{d\lambda}{dt}, \quad \text{or} \quad e = K \frac{d\lambda}{dt}$$

where K is a constant whose value depends on the system of units chosen. When the change in the linkage is localized in a coil forming part of the circuit, the instantaneous e.m.f. localized in the coil, and, therefore, the e.m.f. of the circuit, is likewise

$$e \propto \frac{d\lambda}{dt}, \quad \text{or} \quad e = K \frac{d\lambda}{dt}$$

If the resistance of the circuit is constant and equal to R , a current will be set up whose instantaneous value i is

$$i = \frac{e}{R}$$

and the quantity of electricity dq flowing in a time dt through any section of the conductor is

$$dq = i dt = \frac{e}{R} dt = K \frac{d\lambda}{R}$$

For a finite change in the linkage, $\lambda_2 - \lambda_1$, the total quantity of electricity Q discharged through the circuit is *independent* of the

time, as is shown by the following relation:

$$Q = \int_{\lambda_1}^{\lambda_2} K \frac{d\lambda}{R} = K \frac{\lambda_2 - \lambda_1}{R}$$

When R is in electromagnetic units (abohm) and Q is in e.m.u. (abcoulomb), $K = 1$ and

$$Q = \frac{\lambda_2 - \lambda_1}{R} \text{ abcoulombs} \quad (1)$$

The flux ϕ of lines of magnetic induction for a long and closely wound solenoid is given near the center by

$$\phi = BA = 4\pi i n_1 \mu A \text{ maxwells} \quad (2)$$

where i is the current in abamperes, n_1 the number of turns *per centimeter length* of solenoid, μ is the magnetic permeability of the solenoid core relative to air, A is the cross-section of the core in square cms., and B is the flux density of lines of magnetic induction in gauss.

If a secondary coil with a total number of turns, N_2 , is wound around the central portion of the solenoid, the *number of linkages* with the *secondary circuit* for a current of i abamperes in the primary is very approximately

$$\lambda = 4\pi i n_1 N_2 \mu A \quad (3)$$

If this secondary coil of N_2 turns forms part of a closed electrical circuit of constant resistance R , the quantity of electricity in e.m.u. discharged through the secondary circuit whenever the primary current changes from i_1 to i_2 is by equations (1) and (3)

$$Q = \frac{4\pi n_1 N_2 A}{R} (\mu_2 i_2 - \mu_1 i_1) = \frac{\lambda_2 - \lambda_1}{R} \quad (4)$$

For ferromagnetic substances μ depends on the value of the magnetizing current. For air, it is constant and equal to unity.

The deflections of a ballistic galvanometer in the secondary circuit will be proportional to the quantity of electricity induced provided the time of the induction is very short compared with the half-period of the galvanometer. Consequently, provided the resistance of the secondary circuit is constant, the galvanometer deflections will also be proportional to the change in the

number of linkages ($\lambda_2 - \lambda_1$) through the secondary of the solenoid, which in turn is proportional to the change in the magnetic flux ϕ of the solenoid near its center.

Instead of producing a change in the number of linkages of the secondary circuit by varying the current in the primary of the solenoid, the *test coil* may be suddenly removed from a magnetic field, causing a deflection of the ballistic galvanometer. Provided the dimensions and number of turns of the test coil are known and the magnetic field in which it is placed is uniform, the intensity of this magnetic field may be compared with that of the solenoid having an air core.

Apparatus.—A ballistic d'Arsonval galvanometer; a test or flip coil of known dimensions; a standard straight solenoid having a secondary coil of a known number of turns; suitable rheostats; an ammeter; switches and a 120-volt storage battery; an electromagnet with parallel pole faces.

Manipulation, Computations and Plots.—PART I.—Calibration of the ballistic galvanometer when in series with the test or flip coil and the secondary of the standard solenoid.

Connect the ballistic galvanometer, test coil and secondary of the standard solenoid in series to form a closed circuit. See Fig. 28. Connect the terminals of the solenoid proper through a rheostat, switch and ammeter to the terminals of a 120-volt storage-battery circuit. The solenoid is provided with a snap switch to make and break the circuit suddenly.

Use about six different values of the current in the primary to give deflections on the galvanometer scale over a range from *one* to *twenty* centimeters. Record the deflections on opening and on closing the solenoid circuit *four times* for *each value* of the *current*. Note the direction of the deflections, whether on the red or the black scale.

Using the average deflection obtained for each value of the current upon opening the circuit, plot a curve between galvanometer deflections as ordinates and change in linkages of the galvanometer circuit as abscissas. (Compute the change in linkages by means of formula (3), using the values for the constants of the circuit given in the appended table.) This curve should be plotted to such a scale that it may be used with a precision of at least 1 per cent. at a value of 10 cm. deflection.

PART II.—The object of this part of the exercise is to study the

magnetic behavior of a specimen of iron in the form of a hollow square with a narrow air gap.

The specimen of iron is wound with magnetizing coils, and the faces of the air gap are so large that the magnetic field near the center of the gap may be considered uniform.

The test or flip coil, connected to the galvanometer circuit exactly as in Part I, is placed at the center of the air gap with its plane perpendicular to the magnetic field. By a simple device, the flip coil can be jerked from the gap to a region where there is no magnetic field. The magnetizing coils are connected in series with the ammeter, rheostat, switch and 120-volt terminals used with the solenoid in Part I.

For each value of the magnetizing current, jerk the flip coil out of the field *two* times, recording each time the magnitude and direction of the galvanometer deflection. The deflections should be in the same direction as those obtained in breaking the circuit in Part I. Do *not break the circuit between the changes of current* but proceed with the currents in the following order: 0, 0.5, 1.0, 1.5, 2.0, 1.5, 1.0, 0.5, 0, — 0.5, — 1.0, — 1.5, — 2.0, — 1.5, — 1.0, — 0.5, 0 amperes.

Plot the results with galvanometer deflections as ordinates and magnetizing currents as abscissas.

Note.—Since the flip coil circuit is identical with the one used in Part I to calibrate the galvanometer, the curve of deflections of Part I may be used to determine the change in the number of linkages in Part II. The dimensions of the flip coil being known (see table), the value of B can be computed for any desired field in the air gap.

Problem.—Compute B in gausses for the field in the gap when the current through the magnetizing coils is 2 amperes. The experimental data together with the dimensions of the test coil are to be used in the solution.

TABLE OF CONSTANTS.

Standard Solenoid:

Primary Winding:

Total number of turns.....	400
Total length of solenoid.....	93.82 cm.
Effective cross-sectional area	15.72 cm. ²

Secondary Winding.

Total number of turns.....	1000
----------------------------	------

Test Coil (rectangular):

Total number of turns.....	5
Length.....	10 cm.
Width.....	1 cm.

EXERCISE 36

FORCE ON A CONDUCTOR CARRYING A CURRENT IN A MAGNETIC FIELD

Object.—To verify the numerical relation of the mechanical force on a wire conductor to the current in the conductor, the length of the conductor and the flux density of lines of magnetic induction of the magnetic field in which the conductor is placed, or, in other words, to verify the relation

$$F = kIBl \sin \theta$$

where k is a constant whose value depends on the units employed.

General Discussion.—Ampere (1821) made the very important discovery that a conductor carrying a current, when introduced into a magnetic field, is acted upon by a force tending to move it across the field. This force is due to the mutual action between the magnetic field of the conductor produced by the current and the original magnetic field. The mechanical force df on a small element dl of the conductor due to a current I when the angle between the conductor and the positive direction of the magnetic field is θ was shown by Ampere to be

$$df \propto IB \, dl \sin \theta$$

where B is the flux density of lines of magnetic induction. If the conductor is a straight wire lying at right angles to a uniform magnetic field of flux density $B = \mu H$, this equation becomes by integration

$$F \propto IBl$$

or

$$F = kIBl \quad (1)$$

where k is a constant. Here F is the total force on the conductor tending to move it in a direction *perpendicular* to the *field* and to the *direction* of the *current*. A very useful rule to determine the direction of the force when the direction of the magnetic field and the current are known is the left-hand rule. If the thumb, first and second fingers of the *left* hand are held at right angles to one another, the first or index finger pointing in the positive

direction of the magnetic field in which the conductor is placed, while the second finger is made to point in the direction of the current, then the thumb will point in the direction of the force.

If the medium in which the straight conductor is placed is air, equation (1) may be written

$$F = kIHl \quad (2)$$

where H is the intensity of the magnetic field in dynes per unit pole, or, from the standpoint of lines of force, in gaussess. This equation may serve to define the c.g.s. electromagnetic unit of current as the current which, flowing through a wire conductor placed at right angles to a uniform magnetic field of unit strength, the gauss, experiences a force of one dyne on each centimeter length of the conductor. For the c.g.s. electromagnetic system of units, therefore, k equals unity.

References: Brooks and Poyser, Chap. XVII, and pp. 343-348.

Apparatus.—An equal-arm balance of special design, the wire conductor under investigation being at the extremity of one of the arms, and a set of weights for determining F ; the iron core with magnetizing coils used in Exercise 35, the pole faces of the narrow air gap being so large that the magnetic field of the gap except near the edges is uniform; two ammeters; storage cells for supplying current to the conductor; a 120-volt storage battery for the magnetizing coils; reversing switch; rheostats; flip coil; standardized ballistic galvanometer.

Manipulation.—Connect the apparatus as indicated in the accompanying diagram. By means of the flip coil measure the *residual* magnetic field intensity in the air gap, taking two readings. Note the directions of the galvanometer deflection and make it the same as that on the break of the solenoid current in Exercise 35.

Place the balance in position with the conductor in the center of the narrow air gap of the electromagnet and adjust the stops so that the pointer cannot vibrate more than two or three divisions to either side of the zero point. If this pointer does not come to rest on the zero of the balance scale, adjust the balance by means of the counterbalancing nut on the arm. Add a 300-milligram weight to the balance pan and determine the current required in the wire conductor to balance this weight for the case

of the *residual magnetic field*. The balance may be tapped lightly, if necessary, to insure its free motion. It may be expedient to make two or three determinations of this value in case the balance does not oscillate freely on its bearings. Similar measurements should be made with loads of 500 and 600 milligrams. Remove the balance and take two more readings of the residual field intensity with the flip coil.

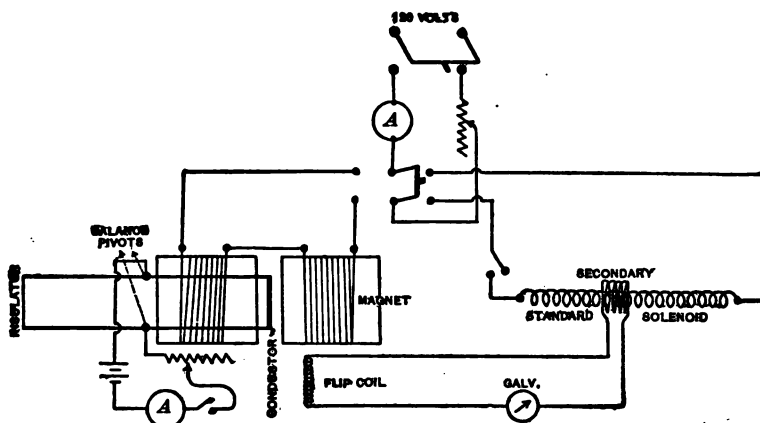


FIG. 28.

Next adjust the rheostats in the magnetizing circuit of the electromagnet to give a current of 0.50 ampere. See that this current is maintained constant, as it is liable to change owing to the increase in the resistance of the magnetizing coils with the heating by the current. Make two determinations of the field intensity corresponding to the magnetizing current of 0.5 ampere. Then place the balance in position and determine, as before, the necessary currents to balance loads of 0.50, 1.00, 1.50 and 2.00 grams. Remove the balance and take two more readings with the flip coil before changing the magnetizing current.

Increase the current in the magnetizing coils to 1.00 ampere. Measure the corresponding intensity of the magnetic field and determine the balancing currents in the conductor for loads of 1, 2, 3, and 4 grams.

Make similar measurements with a magnetizing current of 1.50 amperes and loads of 1, 3, 4 and 5 grams; and finally with a magnetizing current of 2.00 amperes and loads of 1, 3, 5 and 6 grams.

Remove the balance, and with the vernier caliper measure the effective length of the conductor in the magnetic field.

SUGGESTED FORM FOR DATA SHEET.

I	II	III	IV	V
Magnetising Current in Amperes.	<i>Flip Coil</i> Galvanometer Deflections in Millimeters.	Balance Load in Grams.	Balance Current in Amperes.	Averages of Bal- ance Currents.
0.00	== =	0.300 0.500 0.600	== = == = == =	--- --- ---
	== =			
	Av.			
0.50	== =	0.500 1.000 1.500 2.000	== = == = == = == =	--- --- --- ---
	== =			
	Av.			
1.00	== =	1.000 2.000 3.000 4.000	== = == = == = == =	--- --- --- ---
	== =			
	Av.			
1.50	== =	1.000 3.000 4.000 5.000	== = == = == = == =	--- --- --- ---
	== =			
	Av.			
2.00	== =	1.000 3.000 5.000 6.000	== = == = == = == =	--- --- --- ---
	== =			
	Av.			

Plots and Computations.—For each value of the magnetic field, plot a series of curves using balancing currents as ordinates and loads in grams as abscissas, *i. e.*, plot the results of columns

III and V and *label each curve* according to the *deflection reading* in column II. These curves should be straight lines passing through the origin, showing that when B is constant, F is proportional to the current I .

From this set of curves, read off the values of the galvanometer deflections (proportional to B or H) and the loads corresponding to a constant value of the balancing currents, and plot these values for the balancing currents 0.5, 1.0, 1.5 and 2.0 amperes. This should give four straight lines showing that when I is constant F is proportional to B .

Using the first set of curves, pick out at least four points on each line and determine the values in gaussses,* abamperes and dynes corresponding to each point. Then compute in each case the value for l , the length of the conductor that is at right angles to the field, by means of the relation

$$l = \frac{F}{IB}$$

Determine the A.D. for this set of values of l . What per cent. is it of the value of l obtained by the vernier caliper?

* The actual value of the field intensities, B , should be obtained by means of the galvanometer curve of Exercise 35 and the dimensions of the flip coil given in the Table of Constants. See problem at end of Exercise 35.

CONSTANTS

$$g = 980.25 \text{ cm./sec.}^2 = 32.160 \text{ ft./sec.}^2 \quad (\text{Columbia})$$

Longitude 4 hrs., 55 min., 50 sec. (Columbia)

Latitude 40° 48' 27" (Columbia)

$$\tau = 3.14159 \quad \log \tau = .497150.$$
$$\begin{array}{ll} \tau = 9.11109 & \log \tau = .197100. \\ \tau^2 = 9.86960 & \log \tau^2 = .994300 \end{array}$$
$$1 \text{ radian} = 57^\circ 17' 45'' = 57.2958.$$
$$1^\circ = 0.017453 \text{ radian}$$

In most of these constants the last figure is not exact. For the precision measure of these numbers the reader must refer to more complete tables and to original sources.

1 meter = 39.37* inches.	1 inch = 25.4000 mm.
1 meter = 3.28083 ft.	1 foot = 30.4801 cm.
1 meter = 1.09361 yd.	1 yard = 0.914402 m.
1 cm. ² = 0.155000 sq. in.	1 sq. in. = 6.45163 cm. ²
1 m. ² = 10.7639 sq. ft.	1 sq. yd. = 0.836131 m. ²
1. cm. ³ = 0.0610234 cu. in.	1 cu. in. = 16.3872 cm. ³
1 liter = 1.05671 quart	1 cu. yd. = 0.764559 m. ³
1 kg = 2.20462 lb. av.	1 lb. av. = 453.592 g.
1 g = 0.0352740 oz.	1 oz. = 28.3495 g.
1 g = 15.4324 grains	1 lb. av. = 7000 grains.

The mean calorie (gram calorie) is $\frac{1}{100}$ of the heat required to raise the mass of one gram of water from 0°C . to 100°C .

1 mean calorie (g-cal) = 4.183×10^7 ergs = 4.183 joules.

1 mean B.t.u. = 777.5 ft.-lb., where the value of g is taken as 980.665 cms. per sec.²

$$1 \text{ mean calorie (g-cal)} = 3.9683 \times 10^{-6} \text{ mean B.t.u.}$$
$$1 \text{ mean B.t.u.} = 2.5200 \times 10^3 \text{ mean calories.}$$

1 English and American horse-power (h.p.) = 746 watts or 550 foot-pounds

* **Exact by definition.**

per second at 50° latitude and sea-level, or 550.22 ft.-lb. per second corresponding to $g = 980.665 \frac{\text{cm.}}{\text{sec.}^2}$

1 kw. = 1.3404 h.p.

Heat of vaporization of water at 100° C. = 539.1 g. cal. per gram.
= 970.4 B.t.u. per lb.

Heat of fusion of ice = 79.8 g.-cal per gram.
= 143.6 B.t.u. per lb.

Mass of 1 cu. ft. of water at 60° F. = 62.3667 lbs.

1 Standard atmosphere of pressure (by definition) = 760 mm. of mercury at 0° C. and latitude 45° at sea-level. = 29.921 in. of mercury = 14.696 lb. per sq. in.

CONSTANTS FOR WATER.

Temperature, ° C.	Sp. Gravity Referred to Water at 4°.	Sp. Heat* in Terms of 15° Calorie.	Saturated Vapor Pressure mm. of Mercury.
0	0.999868	1.0091	4.57
4	1.000000		6.07
5	0.999992	1.0050	6.51
10	727	1.0020	9.14
15	126	1.0000	12.67
20	0.998230	0.9987	17.36
25	.997071	.9978	23.52
30	.995673	.9973	31.51
35	.994058	.9971	41.78
40	.99224	.9971	54.87
45	.99025	.9973	71.36
50	.98807	.9977	91.98
60	.98324	.9988	148.88
70	.97781	1.0001	233.31
80	.97183	1.0014	354.87
90	.96534	1.0028	525.47
100	.95838	1.0043	760.00

CONSTANTS FOR MERCURY.

Temperature, ° C.	Sp. Gravity	Sp. Heat* in Terms of 15° Calorie.	Saturated Vapor Pressure mm. of Mercury.
0	13.5955	.03346	
10	13.5708	.03335	
20	13.5462	.03325	
30	13.5217	.03316	0.003
40	13.4973	.03308	
50	13.4729	.03300	0.01
60	13.4486	.03294	
70	13.4244	.03289	0.04
80	13.4003	.03284	
90	13.3762	.03277	
100	13.3522	.03269	0.27

* Barnes's results for specific heat of water.

TABLE OF MELTING POINTS.

(BUREAU OF STANDARDS, 1915.)

Helium.....	< - 271° C.	Aluminum.....	658.7
Hydrogen.....	- 259	Silver.....	960.5
Nitrogen.....	- 210	Gold.....	1063.0
Mercury.....	- 39	Copper.....	1083.0
Tin.....	231.9	Nickel.....	1452
Zinc.....	419.4	Iron.....	1530
Antimony.....	630.0	Palladium.....	1549
Platinum.....1755			
Tungsten.....3297 (Langmuir)			

TABLE OF BOILING POINTS.

Water.....	C.° = 100 + 0.03670	(H - 760)
Naphthalene.....	C. = 217.96 + 0.058	(H - 760)
Benzophenone.....	C. = 305.9 + 0.063	(H - 760)
Sulphur.....	C. = 444.6 + 0.0908	(H - 760)

HEATS OF COMBUSTION.

Sucrose.....	3,950	calories per gram.
Benzoic acid.....	6,330	" " "
Naphthalene.....	9,620	" " "

TABLE
FOUR PLACE LOGARITHMS.

LOGS. 4 PL.

No.	0	1	2	3	4	5	6	7	8	9	Interpolation Tables.
1.00	.0000	.0004	.0009	.0013	.0017	.0022	.0026	.0030	.0035	.0039	38 36 34 32
.01	.0043	.0048	.0052	.0056	.0060	.0065	.0069	.0073	.0077	.0082	4 4 3 3
.02	.0086	.0090	.0095	.0099	.0103	.0107	.0111	.0116	.0120	.0124	8 7 7 6
.03	.0128	.0133	.0137	.0141	.0145	.0149	.0154	.0158	.0162	.0166	11 11 10 10
.04	.0170	.0175	.0179	.0183	.0187	.0191	.0195	.0199	.0204	.0208	15 14 14 13
1.05	.0212	.0216	.0220	.0224	.0228	.0233	.0237	.0241	.0245	.0249	19 18 17 16
.06	.0253	.0257	.0261	.0265	.0269	.0273	.0278	.0282	.0286	.0290	23 22 20 19
.07	.0294	.0298	.0302	.0306	.0310	.0314	.0318	.0322	.0326	.0330	27 25 24 22
.08	.0334	.0338	.0342	.0346	.0350	.0354	.0358	.0362	.0366	.0370	30 29 27 26
.09	.0374	.0378	.0382	.0386	.0390	.0394	.0398	.0402	.0406	.0410	34 32 31 29
1.0	.0000	.0043	.0086	.0128	.0170	.0212	.0253	.0294	.0334	.0374	30 28 26 24
.1	.0414	.0453	.0492	.0531	.0569	.0607	.0645	.0682	.0719	.0755	3 3 3 2
.2	.0792	.0828	.0864	.0899	.0934	.0969	.1004	.1038	.1072	.1106	6 6 5 5
.3	.1139	.1173	.1206	.1239	.1271	.1303	.1335	.1367	.1399	.1430	9 8 8 7
.4	.1461	.1492	.1523	.1553	.1584	.1614	.1644	.1673	.1703	.1732	12 11 10 10
1.5	.1761	.1790	.1818	.1847	.1875	.1903	.1931	.1959	.1987	.2014	15 14 13 12
.6	.2041	.2068	.2095	.2122	.2148	.2175	.2201	.2227	.2253	.2279	18 17 16 14
.7	.2304	.2330	.2355	.2380	.2405	.2430	.2455	.2480	.2504	.2529	21 20 18 17
.8	.2553	.2577	.2601	.2625	.2648	.2672	.2695	.2718	.2742	.2765	24 22 21 19
.9	.2788	.2810	.2833	.2856	.2878	.2900	.2923	.2945	.2967	.2989	27 25 23 22
2.0	.3010	.3032	.3054	.3075	.3096	.3118	.3139	.3160	.3181	.3201	22 20 18 16
.1	.3222	.3243	.3263	.3284	.3304	.3324	.3345	.3365	.3385	.3404	2 2 2 2
.2	.3424	.3444	.3464	.3483	.3502	.3522	.3541	.3560	.3579	.3598	4 4 4 3
.3	.3617	.3636	.3655	.3674	.3692	.3711	.3729	.3747	.3766	.3784	7 6 5 5
.4	.3802	.3820	.3838	.3856	.3874	.3892	.3909	.3927	.3945	.3962	9 8 7 6
2.5	.3979	.3997	.4014	.4031	.4048	.4065	.4082	.4099	.4116	.4133	11 10 9 8
.6	.4150	.4166	.4183	.4200	.4216	.4232	.4249	.4265	.4281	.4298	13 12 11 10
.7	.4314	.4330	.4346	.4362	.4378	.4393	.4409	.4425	.4440	.4456	15 14 13 11
.8	.4472	.4487	.4502	.4518	.4533	.4548	.4564	.4579	.4594	.4609	18 16 14 13
.9	.4624	.4639	.4654	.4669	.4683	.4698	.4713	.4728	.4742	.4757	20 18 16 14
3.0	.4771	.4786	.4800	.4814	.4829	.4843	.4857	.4871	.4886	.4900	15 14 13 12
.1	.4914	.4928	.4942	.4955	.4969	.4983	.4997	.5011	.5024	.5038	2 1 1 1
.2	.5051	.5065	.5079	.5092	.5105	.5119	.5132	.5145	.5159	.5172	3 3 3 2
.3	.5185	.5198	.5211	.5224	.5237	.5250	.5263	.5276	.5289	.5302	5 4 4 4
.4	.5315	.5328	.5340	.5353	.5366	.5378	.5391	.5403	.5416	.5428	6 6 5 5
3.5	.5441	.5453	.5465	.5478	.5490	.5502	.5514	.5527	.5539	.5551	8 7 7 6
.6	.5563	.5575	.5587	.5599	.5611	.5623	.5635	.5647	.5658	.5670	9 8 8 7
.7	.5682	.5694	.5705	.5717	.5729	.5740	.5752	.5763	.5775	.5786	11 10 9 8
.8	.5798	.5809	.5821	.5832	.5843	.5855	.5866	.5877	.5888	.5899	12 11 10 10
.9	.5911	.5922	.5933	.5944	.5955	.5966	.5977	.5988	.5999	.6010	14 13 12 11
4.0	.6021	.6031	.6042	.6053	.6064	.6075	.6085	.6096	.6107	.6117	11 10 9 8
.1	.6128	.6138	.6149	.6160	.6170	.6180	.6191	.6201	.6212	.6222	1 1 1 1
.2	.6232	.6243	.6253	.6263	.6274	.6284	.6294	.6304	.6314	.6325	2 2 2 2
.3	.6335	.6345	.6355	.6365	.6375	.6385	.6395	.6405	.6415	.6425	3 3 3 2
.4	.6435	.6444	.6454	.6464	.6474	.6484	.6493	.6503	.6513	.6522	4 4 4 3
4.5	.6532	.6542	.6551	.6561	.6571	.6580	.6590	.6599	.6609	.6618	6 5 5 4
.6	.6628	.6637	.6646	.6656	.6665	.6675	.6684	.6693	.6702	.6712	7 6 5 5
.7	.6721	.6730	.6739	.6749	.6758	.6767	.6776	.6785	.6794	.6803	8 7 6 6
.8	.6812	.6821	.6830	.6839	.6848	.6857	.6866	.6875	.6884	.6893	9 8 7 6
.9	.6902	.6911	.6920	.6928	.6937	.6946	.6955	.6964	.6972	.6981	10 9 8 7

LOGS. 4 PL.

FOUR PLACE LOGARITHMS.

4 PL. LOGS.

No.	0	1	2	3	4	5	6	7	8	9	Interpolation Tables.
5.0	.6990	.6998	.7007	.7016	.7024	.7033	.7042	.7050	.7059	.7067	9 8 7
.1	.7076	.7084	.7093	.7101	.7110	.7118	.7126	.7135	.7143	.7152	1 1 1
.2	.7160	.7168	.7177	.7185	.7193	.7202	.7210	.7218	.7226	.7235	2 2 1
.3	.7243	.7251	.7259	.7267	.7275	.7284	.7292	.7300	.7308	.7316	3 2 2
.4	.7324	.7332	.7340	.7348	.7356	.7364	.7372	.7380	.7388	.7396	4 3 3
5.5	.7404	.7412	.7419	.7427	.7435	.7443	.7451	.7459	.7466	.7474	5 4 4
.6	.7482	.7490	.7497	.7505	.7513	.7520	.7528	.7536	.7543	.7551	5 5 4
.7	.7559	.7566	.7574	.7582	.7589	.7597	.7604	.7612	.7619	.7627	6 6 5
.8	.7634	.7642	.7649	.7657	.7664	.7672	.7679	.7686	.7694	.7701	7 6 6
.9	.7709	.7716	.7723	.7731	.7738	.7745	.7752	.7760	.7767	.7774	8 7 6
6.0	.7782	.7789	.7796	.7803	.7810	.7818	.7825	.7832	.7839	.7846	7 6
.1	.7853	.7860	.7868	.7875	.7882	.7889	.7896	.7903	.7910	.7917	1 1
.2	.7924	.7931	.7938	.7945	.7952	.7959	.7966	.7973	.7980	.7987	1 1
.3	.7993	.8000	.8007	.8014	.8021	.8028	.8035	.8041	.8048	.8055	2 2
.4	.8062	.8069	.8075	.8082	.8089	.8096	.8102	.8109	.8116	.8122	3 2
6.5	.8129	.8136	.8142	.8149	.8156	.8162	.8169	.8176	.8182	.8189	4 3
.6	.8195	.8202	.8209	.8215	.8222	.8228	.8235	.8241	.8248	.8254	4 4
.7	.8261	.8267	.8274	.8280	.8287	.8293	.8299	.8306	.8312	.8319	5 4
.8	.8325	.8331	.8338	.8344	.8351	.8357	.8363	.8370	.8376	.8382	6 5
.9	.8388	.8395	.8401	.8407	.8414	.8420	.8426	.8432	.8439	.8445	6 5
7.0	.8451	.8457	.8463	.8470	.8476	.8482	.8488	.8494	.8500	.8506	6 5
.1	.8513	.8519	.8525	.8531	.8537	.8543	.8549	.8555	.8561	.8567	1 1
.2	.8573	.8579	.8585	.8591	.8597	.8603	.8609	.8615	.8621	.8627	1 1
.3	.8633	.8639	.8645	.8651	.8657	.8663	.8669	.8675	.8681	.8686	2 2
.4	.8692	.8698	.8704	.8710	.8716	.8722	.8727	.8733	.8739	.8745	2 2
7.5	.8751	.8756	.8762	.8768	.8774	.8779	.8785	.8791	.8797	.8802	3 3
.6	.8808	.8814	.8820	.8825	.8831	.8837	.8842	.8848	.8854	.8859	4 3
.7	.8865	.8871	.8876	.8882	.8887	.8893	.8899	.8904	.8910	.8915	4 4
.8	.8921	.8927	.8932	.8938	.8943	.8949	.8954	.8960	.8965	.8971	5 4
.9	.8976	.8982	.8987	.8993	.8998	.9004	.9009	.9015	.9020	.9025	5 5
8.0	.9031	.9036	.9042	.9047	.9053	.9058	.9063	.9069	.9074	.9079	6 5
.1	.9085	.9090	.9096	.9101	.9106	.9112	.9117	.9122	.9128	.9133	1 1
.2	.9138	.9143	.9149	.9154	.9159	.9165	.9170	.9175	.9180	.9186	1 1
.3	.9191	.9196	.9201	.9206	.9212	.9217	.9222	.9227	.9232	.9238	2 2
.4	.9243	.9248	.9253	.9258	.9263	.9269	.9274	.9279	.9284	.9289	2 2
8.5	.9294	.9299	.9304	.9309	.9315	.9320	.9325	.9330	.9335	.9340	3 3
.6	.9345	.9350	.9355	.9360	.9365	.9370	.9375	.9380	.9385	.9390	4 3
.7	.9395	.9400	.9405	.9410	.9415	.9420	.9425	.9430	.9435	.9440	4 4
.8	.9445	.9450	.9455	.9460	.9465	.9469	.9474	.9479	.9484	.9489	5 4
.9	.9494	.9499	.9504	.9509	.9513	.9518	.9523	.9528	.9533	.9538	5 5
9.0	.9542	.9547	.9552	.9557	.9562	.9566	.9571	.9576	.9581	.9586	5 4
.1	.9590	.9595	.9600	.9605	.9609	.9614	.9619	.9624	.9628	.9633	1 0
.2	.9638	.9643	.9647	.9652	.9657	.9661	.9666	.9671	.9675	.9680	1 1
.3	.9685	.9689	.9694	.9699	.9703	.9708	.9713	.9717	.9722	.9727	2 1
.4	.9731	.9736	.9741	.9745	.9750	.9754	.9759	.9763	.9768	.9773	2 2
9.5	.9777	.9782	.9786	.9791	.9795	.9800	.9805	.9809	.9814	.9818	3 2
.6	.9823	.9827	.9832	.9836	.9841	.9845	.9850	.9854	.9859	.9863	3 2
.7	.9868	.9872	.9877	.9881	.9886	.9890	.9894	.9899	.9903	.9908	4 3
.8	.9912	.9917	.9921	.9926	.9930	.9934	.9939	.9943	.9948	.9952	4 3
.9	.9956	.9961	.9965	.9969	.9974	.9978	.9983	.9987	.9991	.9996	5 4

4 PL. LOGS.

DEGREE	SINE	TANGENT	COTANGENT	COSINE	DEGREE
0	0.0000	0.0000	∞	1.0000	90
1	.0175	.0175	57.29	0.9998	89
2	.0349	.0349	28.64	.9994	88
3	.0523	.0524	19.08	.9986	87
4	.0698	.0699	14.30	.9976	86
5	0.0872	0.0875	11.43	0.9962	85
6	.1045	.1051	9.514	.9945	84
7	.1219	.1228	8.144	.9925	83
8	.1392	.1405	7.115	.9903	82
9	.1564	.1584	6.314	.9877	81
10	0.1736	0.1763	5.671	0.9848	80
11	.1908	.1944	5.145	.9816	79
12	.2079	.2126	4.705	.9781	78
13	.2250	.2309	4.331	.9744	77
14	.2419	.2493	4.011	.9703	76
15	0.2588	0.2679	3.732	0.9659	75
16	.2756	.2867	3.487	.9613	74
17	.2924	.3057	3.271	.9563	73
18	.3090	.3249	3.078	.9511	72
19	.3256	.3443	2.904	.9455	71
20	0.3420	0.3640	2.747	0.9397	70
21	.3584	.3839	2.605	.9336	69
22	.3746	.4040	2.475	.9272	68
23	.3907	.4245	2.356	.9205	67
24	.4067	.4452	2.246	.9135	66
25	0.4226	0.4663	2.145	0.9063	65
26	.4384	.4877	2.050	.8988	64
27	.4540	.5095	1.963	.8910	63
28	.4695	.5317	1.881	.8829	62
29	.4848	.5543	1.804	.8746	61
30	0.5000	0.5774	1.732	0.8660	60
31	.5150	.6009	1.664	.8572	59
32	.5299	.6249	1.600	.8480	58
33	.5446	.6494	1.540	.8387	57
34	.5592	.6745	1.483	.8290	56
35	0.5736	0.7002	1.428	0.8192	55
36	.5878	.7265	1.376	.8090	54
37	.6018	.7536	1.327	.7986	53
38	.6157	.7813	1.280	.7880	52
39	.6293	.8098	1.235	.7771	51
40	0.6428	0.8391	1.192	0.7660	50
41	.6561	.8693	1.150	.7547	49
42	.6691	.9004	1.111	.7431	48
43	.6820	.9325	1.072	.7314	47
44	.6947	.9657	1.036	.7193	46
45	0.7071	1.0000	1.000	0.7071	45
DEGREE	COSINE	COTANGENT	TANGENT	SINE	DEGREE

STANDARD REFERENCE BOOKS.

- U. S. Bureau of Standards: Circulars, Scientific Papers and
Bulletins.
Smithsonian Physical Tables.
Smithsonian Meteorological Tables.
Smithsonian Psychrometric Tables.
Landolt-Boernstein, Physical and Chemical Tables.
Kaye and Laby, Physical and Chemical Constants and some
Mathematical Functions.
Hodgman and Coolbaugh, Handbook of Chemistry and Physics.
Marks and Davis, Steam Tables and Diagrams.
Pender, American Handbook for Electrical Engineers.
Marks, Mechanical Engineering Handbook.
Tracy, Plane Surveying.

